Alpha Power:

Adding More Alpha to Portfolio Return

Abstract

This paper demonstrates that a portfolio manager can improve performance by applying deterministic trading techniques and thereby not only boost but also accelerate the portfolio return attributable to his/her expertise.

From a market model set up from a pre-selected random trading environment to simulate a portfolio composed of stocks with randomly generated price variations, trading procedures under controlled and objective functions are set to implement portfolio management activities. It is shown that the measure of excess return over and above the average market return (Jensen's *alpha*) can be increased substantially by leveraging excess portfolio management skills. The underlying theoretical framework is provided where it is exposed that not only "*alpha*" can be increased but also an *alpha* adjusted Sharpe ratio can increase over time. Even when faced with random price variations with zero expected mean, trading procedures can still be implemented to improve overall performance. It is also shown that one can achieve an increasing exponential Sharpe ratio over time which translates to performance improving in step with deterministic procedures.

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I - Introduction

After numerous testing methods where applied to real historical data, it was found difficult to produce returns that could consistently beat market averages. You would design a trading system that would be optimized over a trading period only to see it fall down when tested forward. The reasons were almost always the same: over optimization, curve fitting and/or knowledge of what had already happened sipping through into the trading strategy's design itself; not counting faulty logic like trading today based on bad data or price data that could only be had tomorrow.

Under the hypothesis that stock prices moved in a quasi-random fashion, it was decided that what ever the trading system designed, it had to provide above average results even when all the data would be randomly generated as if following a random walk. Using randomly generated price series would certainly curtail efforts at over optimization, curve fitting and forecasting.

The purpose of the random test was first to observe, as a group, the behavior of stocks with random variations and with random drift over time; then to find trading procedures, if at all possible, that could produce more than the Buy & Hold over the long haul. The premise was that if real stock market prices act as quasi-random price series, then using totally randomly generated prices should provide most of the same obstacles to making a profit, for any trading procedure, as if in a real market.

This paper is presented upside down in the sense that the results were first obtained and then an explanation was required to better understand the results which in turn led to the need to put it all in the context of portfolio management theory. Therefore, a description of the trading environment is presented first, then the theoretical framework that best describe the results is given followed the test results themselves.

The theoretical framework provided best characterize the results obtained from hundreds of tests done on randomly generated price series in order to simulate a market and a portfolio. Some of the equations to be presented in later sections were the best match for the results obtained and serve as explanation and mathematical expressions that best describe the phenomena under study. The innovation here is in the modified interpretation of the Sharpe ratio which is made to increase with time and thereby increase performance exponentially (up to an undetermined limit).

The objective was not to design yet another procedure to generate random prices; there are sufficient available methods in the literature for this. The one method selected, which should be as good as any other, serves only as backdrop to generate price series at will. The real objective was to design trading procedures that would play on these randomly generated price series in order to outperform the market average – to generate some *alpha*.

From a set of deterministic trading procedures, it is shown that one can not only outperform the Buy & Hold; but improve performance (rising Sharpe ratio) as time goes by; all due to leveraging market expertise and portfolio management skills.

Some notes on the mathematical notation used in this paper.

In an attempt to be concise, every effort has been made to hopefully use the shortest mathematical representation as possible. In this perspective; it was preferred to express equation 1 as is instead of as a stochastic differential equation of the form:

$$dS_i = S_i(t)\,[b_i(t)dt + \Sigma\, \sigma_{ij}(t)dw_j(t)]$$

where the stock price variations (dS_i) *are given as a drift component plus the cumulative sum of variance subjected fluctuations under a generalized Brownian motion. The same goes for expressing the incremental investor's differential wealth at time (t), instead of:*

$$dW(t) = [r(t)W(t) - c(t)]dt + \pi(t)^* [(b(t) - r(t))dt + \sigma(t)dw(t)]$$

where the optimal portfolio $\pi(t)^*$ *could be given as:*

$$\pi(t)^* = \theta^*(t)\, \sigma(t)^{-1} w(t) \quad \text{and where} \quad \theta(t) = \sigma(t)^{-1}[(b(t) - r(t)]$$

the future expected portfolio value $\mathbb{E}(P_v)$ *will be used as presented in equation 7. Both expressions for a portfolio operate in a mean-variance space and have for mean the same expected long term average market return* R_m.

Equations used throughout the text most often will express vector data series starting with an initial value such as P_o *in equation* 1 *and, in this paper, will cover only 1000 weekly periods (i=0,…,1000 about 19.2 years).*

The random drift model was selected as a reasonable surrogate for random price generation which can be easily implemented in Excel. Other models could have been used, as the one above, however the random drift was chosen for its simplicity. The modeled price series will be presented here as follows:

$$\mathbf{P}(t) = P_o + ax + \sum_{t=1}^{t=1000} \varepsilon \qquad (1)$$

where the price **P**(t) will be composed of an initial price (P_o) to which is added the cumulative sum of its random variations (ε) which in turn will be added to its linear regression line (ax) with slope = a, and of length: $x_t - x_o = t$ = 1000 weekly periods. This way all stock price series could be randomly generated; and all that would be required would be to use an almost ready made random component that closely mimicked stock price variations as much as possible. Again, the purpose was not to design a new way to generate random price series, but to use what is already available in random price generation in order to have a random trading environment where you could not use curve fitting, over optimization or forecasting methods and where you could implement what ever trading procedures you wished.

The random variations of equation 1 were made to resemble a Paretian distribution (meaning with fat tails) rather than a Gaussian (normal) distribution as the former better represented stock price movements and idiosyncrasies. It was also decided that the drift component (ax) would be random in nature; this way stocks could have long term random trends up or down.

Obtaining the average price at any time (t) from the set of randomly generated prices would be as easy as dividing their sum by *n*: the number of stocks being part of the test. Equation 2 would serve as the average price at any time (t) for all stocks in the portfolio:

$$\overline{P_t} = \sum_{j=1}^{n} P_j(t) / n = 1/n \sum_{j=1}^{n}(P_{oj} + a_j x_j + \sum_{j=1}^{t} \varepsilon_j) \qquad (2)$$

The average price would also have the same structure as equation 1, namely, a random walk component superimposed over a random drift with average slope: ($1/n\ \Sigma\ a_j$). The average rate of change (slope) would be itself a linear equation representing the average drift for all stocks over the period.

From such a trading environment, trading results were easy to predict. The outcome would be close to this particular market's average long term return and this irrelevant, almost assuredly, what ever you did implement as trading procedures. But it turns out that there is a whole family of procedures of the sub-martingale variety regulated by subordinators (as in a Lévy process) that can transform an expected zero *alpha* into an exponentially increasing one (however, this is going beyond the mathematical expressions I would like to use in this paper). In fact, when looking at the problem from of a long term perspective point of view, it is a whole philosophy of trading procedures with many variations on the same general theme that can be used not only to extract some *alpha* but most importantly to put it on steroids. This has the potential to change the perception of portfolio management at its very core (see equations 11, 4.1, 4.4 and 16).

II - The Test Data

All tests comprised of 50 randomly generated price series over 1000 weekly periods (equivalent to about 19.2 years). Each series was generated with random price variations over a random drift (see equation 1). The random component being of a Paretian type meant that fat tails could occur in any price series; this way mimicking the price movement of price shocks (gaps) in the market more closely. The expected mean variation of the random component, for all series, would have a value of zero: $\mathbf{E}(\mu_j) = 0$, even if its structure was Paretian. While the expected standard deviation $\mathbf{E}(\sigma_j)$ would remain an unknown random variable by design. All random components being uncorrelated, also by design, would result in correlation coefficients approaching zero: $R^2_j = 0$. Even though some notion of the general statistical structure of the trading environment can be described, nothing is available to predict the final value of any price series, and therefore, the final value or terminal wealth of the portfolio will also be an unknown random variable.

For each period, a random price variation was generated for each of the 50 stocks in the portfolio; then trading decisions where taken based on these price variations. For each run, an all new set of price series would be generated. Any price series going down to zero would cancel any share holding in that stock at that time (thereby losing all the money invested in that stock) and would not trade again for the rest of the run - no replacements were allowed. As time went by, fewer stocks would participate in the final results as stocks were dropped from the tradable list. A random number of stocks, up to about 28% (14) of the group, could fail (drop to zero) in any one run. With replacement of failed stocks, results would have been theoretically higher than those presented in later sections.

Totally random price series have no pretensions: they present an unknowable and unpredictable future except in statistical terms (like price variations having a zero expected mean). The random walk theory for real market stock price movements has been in portfolio management textbooks for decades (starting with Bachelier in 1900), however, this paper is not trying to prove or disprove this theory but only trying to show that one can still perform better than the Buy & Hold even if it was in a random walk environment. The random walk theory being one of the main arguments advocated for buying index funds, or for adopting the Buy & Hold strategy.

For every single test run, whatever trading procedure used would be confronted with:

1) unknown random price variations of unknown direction of unknown magnitude,
2) unknown random drift of unknown direction (up or down),
3) unknown future amplitude of price movements,
4) unknown speed (momentum) of price movements,

5) unknown best or worst performers of the group,
6) zero expected mean for the random fluctuations.

On the other hand, you could set trading procedures knowing that:

1) all stock prices were randomly generated,
2) as a group the average drift would be positive (long term secular trend),
3) there was no selection bias possible,
4) you could not win all the time, up to 28% of the group could fail, independent of other trading decisions,
5) you could not predetermine which stock would fail,
6) there could be no survivorship bias possible,
7) there could be no optimization on any individual stock,
8) there could be no curve fitting due to the randomness of every run,
9) there could be no past market knowledge passed on to the next run except maybe the notion that there would be an unknown average positive drift by design (positive average secular trend).

Under these conditions, all you could hope for was a zero sum game, meaning that you should do close to this particular "market" average and that your best strategy was simply a Buy & Hold strategy. Theoretically, you could not win, you could not show any "*alpha*" and thereby condemned to the most probable outcome: the average market rate of return which is simply the slope or the average drift. Your expected outcome would also depend on continuous market exposure; less than full exposure would in all probability mean less than average return.

Even though, after the fact, (meaning after any test run) you could find and trace an "efficient frontier" and determine the perfect portfolio weights; there was no way to determine in advance what that optimum portfolio would or could have been. And since each run provided a new set of 50 series with 1000 randomly generated price variations each, there was no way of forecasting a future optimum portfolio that could reside on the future "efficient frontier". Neither could you use any type of fundamental data: the series being randomly generated. There was no notion even remotely related to fundamental data except maybe the notion that in the long run, the average of all prices would have a tendency to rise (due to the average random positive drift).

The more you ran tests, the more you found that:

1) you could not forecast what was coming up as to price variations,
2) you could not pre-select the best performers as you had no way of knowing which stocks would outperform,
3) you could not preserve yourself from stocks falling to zero,
4) you could not predict how high or how low the prices might go,
5) there was no pricing model that could help you forecast future prices,

6) there was no Sharpe ratio rebalancing that could improve results,
7) there was no profitable pair trading possible,
8) there was no arbitrage trading possible,
9) even though there was correlation between price series, you could not predict which would correlate with which and for how long,
10) past performance was certainly no guarantee of future performance,
11) the more you ran tests, the more it looked like a real market.

For each run, a totally new set of random prices with random amplitude and random drift would be generated. The average long term random drift over the test period was set by design at about 10% per 52 weeks (periods), thereby approaching the average market return with dividend reinvestment (R_m) which is close enough to the real historical market drift. This amounted to less than $0.10 of average upward drift per week (about $0.02 per day). This two cents of average upward drift per day was certainly too small to profit from on a daily basis. It would also be hard to detect such a small drift value from the random price variations: the drift would be drowned in all the random noise.

There was no survivorship bias present since all the stocks that would fail within a run could not be selectively bypassed, ignored or avoided. There was no selection bias either as all 50 stocks of the group were part of the test. Each stock would start with a minimum of 100 shares (trade basis) as initial position with some stocks selected at random having up to a few hundred shares also selected at random.

Naturally, no peeking at future price data or statistical structure was allowed as this would have invalidated the whole research process and would have made this study just another scam. Trading decisions were taken after each period's price change and at the then prevailing price, no exceptions.

The trading procedures, or a combination thereof, had to overcome all obstacles presented, otherwise, the best you could do, on average, would be the equivalent of the Buy & Hold and if that was the case then all this research would be worthless – not quite, at least it would have given another way of not beating the averages. It fast became a quest for "*alpha*" under unknown future price movements in duration, speed, direction and magnitude.

It was surprising, as this research evolved, to find that deterministic trading procedures were changing and enhancing my perception of *alpha* to a point where, I believe, far exceeded previous notions.

III - Testing Using Random Price Series

Testing portfolio management strategies using random stock price series is a lesson in itself. There could be no pretense at stock picking expertise or trading abilities that could prevail from test run to test run. Any "*alpha"* generated would have to be of the pure "luck" variety - a direct consequence of the test's statistical structure. The expected value for "*alpha*" would be zero for every test run.

The worst kind of test for a stock trading strategy is being presented with price series it has never seen; that are uncorrelated and that can have fat tails (following a Paretian distribution): price series which have unknown speed of movement and unknown direction of unknown magnitude. A price series, so random, that it would have a unique and unduplicatable signature. A randomly generated price series can have all these attributes. It can also exhibit cycles, support and resistance levels, trend lines of undetermined length and duration not to mention chart patterns of all types. If your trading procedures can profit from randomly generated prices, then it should be relatively easy for the same procedures to profit from real market data.

The random component of each price series ($\sum \varepsilon_j$) has for expected mean value of zero. Its expected long term rate of change is also zero due to its correlation coefficient, R^2_j, being close to zero. However, the slope of the drift component (see equation 1) will show very high auto-correlation (a straight line). The signal (drift) to noise ratio (random variations) will however be relatively small.

IV - The Theory Behind the Trading Procedures

Once, *alpha* was some times defined as the risk premium which was the return achieved above the risk free rate (r_f) and was expressed in mathematical form as:

$$\mathbf{E}(R_p) = r_f + \beta[\mathbf{E}(R_m) - r_f] \qquad (3)$$

That is, a portfolio's expected return $\mathbf{E}(R_p)$ equals the risk-free rate (r_f) plus the portfolio's beta (β) multiplied by the expected excess return ($\mathbf{E}(R_m) - r_f$) of the market portfolio.

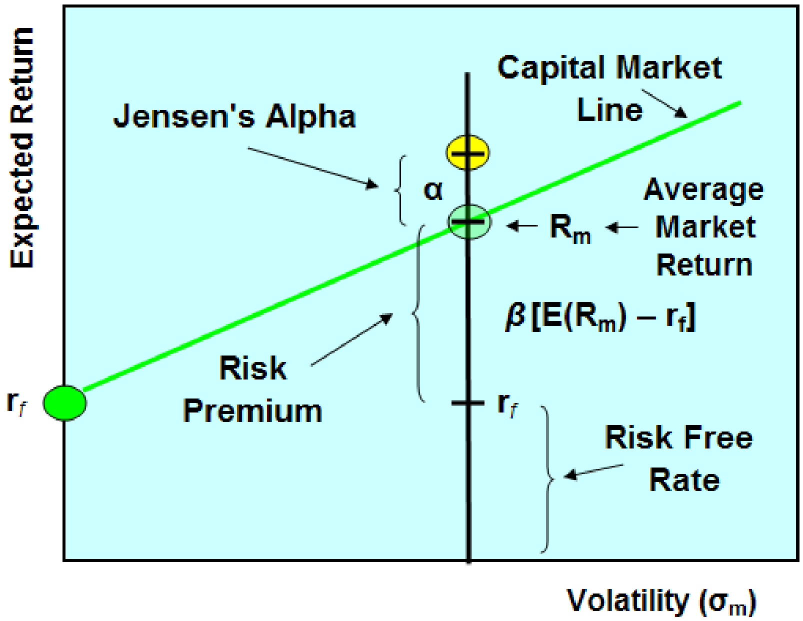
Then, in 1968 Jensen added an *alpha* term:

$$\mathbf{E}(R_p) = r_f + \beta[\mathbf{E}(R_m) - r_f] + \alpha \qquad (4)$$

By adding *alpha* (α), Jensen considered the possibility of a portfolio residing above the Capital Market Line due to trading skills, privileged information or intuition of the portfolio manager (see Figure 1).

It still meant that the expected portfolio return would be proportional to the risk taken but that somehow an extra return could be achieved either by luck or sheer talent at managing a portfolio. Jensen's *alpha* also became a way to measure the over performance of a manager or the relative performance of different portfolios. Depending on the capital under management, even only a few points of added *alpha* could be considered as a big edge over the average market return over a long term investment period and would warrant high bonuses be paid to the portfolio manager for his over performance.

Figure 1: **Jensen's Alpha**



By adding *alpha* to the expected return equation, Jensen was giving a measurable value to the portfolio manager's skills.

The average expected market return (R_m) follows the Capital Market Line (see Figure 1); as risk (volatility) increases, so does the expected portfolio return ($\mathbb{E}(R_p)$). Jensen's *alpha* represents the excess return over and above the average expected market return and is a measure of the portfolio manager's over achievement in risk return space.

In the case of an indexed or a highly diversified portfolio – where beta (β) could equal one – equation 4 would reduce to:

$$\mathbb{E}(R_p) = r_f + \mathbb{E}(R_m) - r_f + \alpha \qquad (5)$$

All that equation 5 promises is that a totally "diversified" portfolio will achieve the market expected return (simply by participation) should the portfolio manager not be able to show above average management skills (*alpha* = 0), then equation 5 with no alpha would result in:

$$\mathbb{E}(R_p) = \mathbb{E}(R_m) \qquad (6)$$

This is an unambiguous promise; it states that on average, the most probable outcome, long term, is to achieve average market returns. It does not matter how you trade or how often, nor why you trade; simply participating in the game, full time, even playing randomly, will somehow provide you with expected results close to the expected long term market average. To achieve more, you need some form of skill and/or luck (*alpha*). Call it having a trading edge or what ever, but without *alpha* your most probable outcome would still remain the market's average secular trend (equation 6).

The US average market return (R_m) over the past 200 years has stood at about 10% including dividend reinvestment with a standard deviation (σ_m) of about 16% and a risk free rate (r_f) of about 3.5%. This translates to a market historical Sharpe ratio (SR) of about 0.406; not that great a number.

$$SR = \frac{R_m - r_f}{\sigma_m} = \frac{0.10 - 0.035}{0.16} = 0.406$$

Achieving the highest possible *alpha* should then be the main task of any portfolio manager with the added constraint of not adding undue risk in the process. In this single number (α) can be expressed the manager's ability to outperform the average. A portfolio manager can use any tool he wants, at any level he wants, to gain *alpha*: be it by better trading techniques, better forecasting methods or even a better understanding of the game. The task remains the same: gaining *alpha* has been the only way to exceed the market average. It is worth noting that about 75% of professional money managers fail to beat the market having an average negative *alpha* of -1.1% (see Jensen 1968).

V - Jensen's Alpha Reformulated

The Jensen expected portfolio performance can be re-expressed as a simple power function such as in the following:

$$\mathbb{E}(P_v) = C_o\,(1 + R_m + \alpha)^t \qquad (7)$$

where $\mathbb{E}(P_v)$ is the expected portfolio value at time (t) and (R_m) the average market return - being defined as before as the result of the risk free rate plus the risk premium ($r_f + \beta[\mathbb{E}(R_m) - r_f]$) - see Figure 1. C_o being the capital outlay used to establish initial stock positions. As in equation 5, *alpha* (α) is the average excess return over and above the average expected market return (R_m).

Without *alpha*, equation 7 is simply another expression for the Buy & Hold. It is your skills, trading abilities and/or "luck" that will determine your overall return either above of below the market average. It is what you will do in the market, your skills or lack of them which, in the end, will make a difference.

Understandably, there is nothing you can do to modify the market's long term average return. The only place where you can claim some form of control is over *alpha* which is also the way to quantify the value of your "skills". *Alpha* can be generated in several ways: better stock picking than average, better trade timing, better allocation, better data mining, better price anomaly detection, better statistical inference, better position sizing, better trading techniques or a combination of all these and more. Separation between luck and skill is often a blurred line when one considers that the majority of market players fail to show a positive *alpha*. But what ever the origin of your *alpha*, it is required if you wish to achieve returns above the Buy & Hold strategy (equation 7).

All one does trading is trying to achieve returns better than the Buy & Hold using all available analytical skills and market knowledge. You can easily imagine that every money manager will also use every possible tool at his/her disposal to achieve the highest possible *alpha*; this even if most will still miss the mark (see Jensen and many others on this subject).

More control is required; it is needed to go beyond *alpha* to raise performance higher. What ever the skill level, performance can be increased by applying additional procedures to existing trading methods. One can not count on luck alone to outperform; one has to implement trading strategies knowing in advance that they will produce better results. This is where a different view of *alpha* is required and therefore, I introduce the:

VI - Alpha Booster

Since we can only improve on *alpha*, we can add a new term to equation 7:

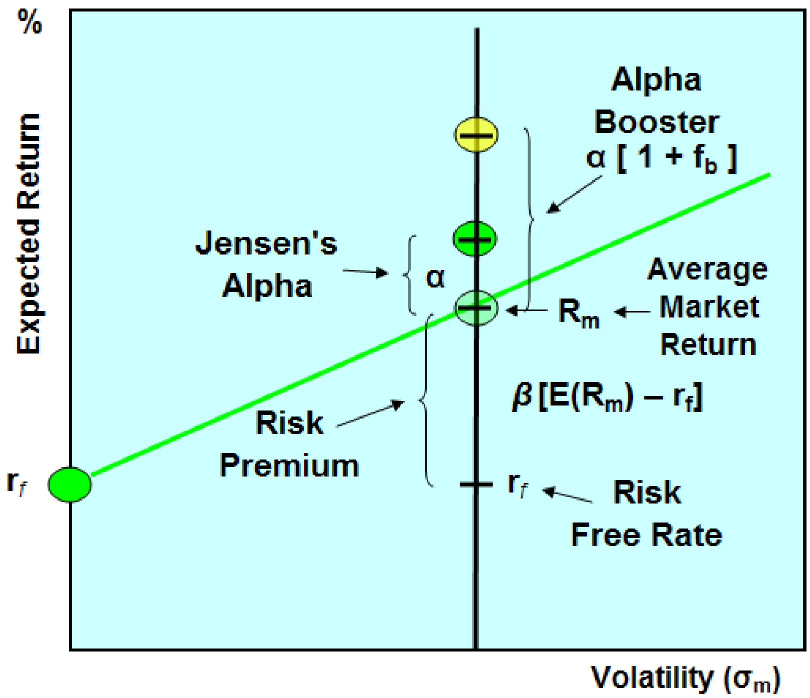
$$\mathbb{E}(P_v) = C_o\,(1 + R_m + \alpha\,[\,1 + f_b\,]\,)^t \qquad (8)$$

where ($[\,1 + f_b\,]$) is the Alpha Booster (AB). Making the parameter (f_b) equal zero will revert equation 8 to the Jensen formula as it should. This incremental factor (f_b), when positive, will improve overall long term performance.

The implication of having an *alpha* booster introduces an added level of skill above the Jensen *alpha* that can be a measure of the portfolio manager's degree of market aggressiveness and acumen knowing that he/she can outperform the market average. Equation 8, not only asserts portfolio management skills, it also states that these skills can be magnified in a controllable fashion. The AB can be a complex equation and can be expressed in the form of deterministic trading procedures. It also means that one can exceed the probable outcome (average market return) by more than *alpha* by using controlled trading procedures which have for only objective to boost returns higher (see Figure 2). To outperform, it then becomes a question of leveraging trading abilities.

Naturally, a negative AB (meaning an $f_b < 0$) will undermine the portfolio manager's efforts to outperform and reduce his/her potential *alpha* and thereby the overall performance. It is therefore required to design trading strategies that can implement and maintain a positive AB ($f_b > 0$).

Figure 2: **Alpha Booster**



Building on the Jensen formula, the *Alpha* Booster (AB) can push performance to a higher level by leveraging portfolio management skills.

At first glance, having an *alpha* booster might seem like having minimal impact on any trading strategy. However when put in a long term perspective, a 3 or 5 points of extra *alpha* over a 20 year period or longer can make a major difference on final results.

Even though the Alpha Booster is interesting in its own right, it already has been superseded by its higher level sibling, namely:

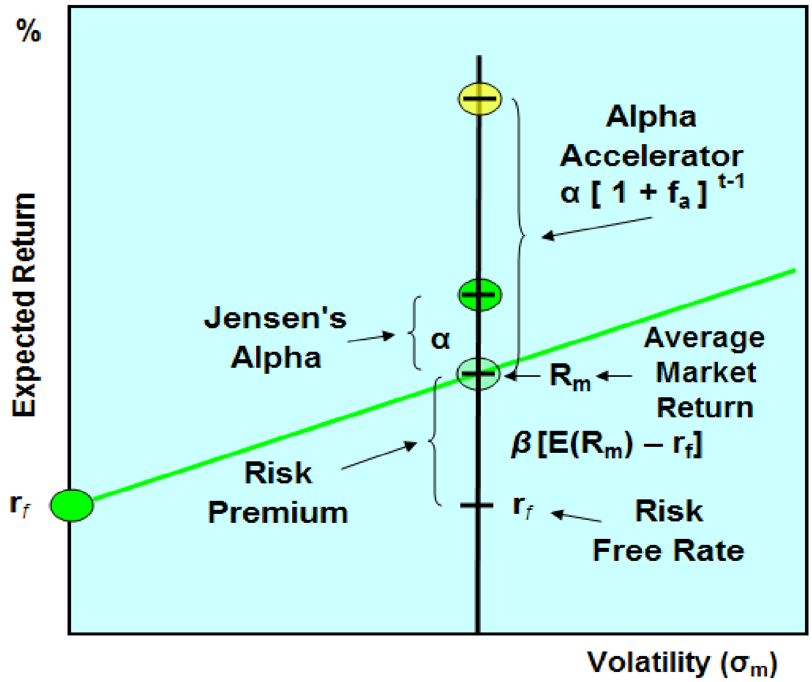
VII - Alpha Accelerator

While the AB adds value to the existing *alpha*, it can still be increase further by using the Alpha Accelerator (AA). In mathematical form, it translates to:

$$P_v = C_o \left(1 + R_m + \alpha \left[1 + f_a \right]^{t-1} \right)^t \qquad (9)$$

where ($[1 + f_a]^{t-1}$) is the Alpha Accelerator. Making the parameter (f_a) equal zero will also revert equation 9 to the Jensen formula, whereas, having an *alpha* of zero, will reduce equation 9 to the Buy & Hold equation. This accelerator implies that *alpha* is increasing as a delayed power function, and therefore, the measure of the level of skill will be improving over time (Figure 3).

Figure 3: **Alpha Accelerator**



The Alpha Accelerator (AA) can increase portfolio management skills (alpha) over time by using deterministic trading procedures.

As shown in Figures 2 and 3, (equations 8 and 9 respectively) *alpha* can be improved; first by the AB which simply jumpstarts the skill level and second by the AA which will not only jumpstart management skills but will also increase its value as time goes by. Having an *alpha* accelerator renders a simple booster obsolete, even if it had merit, it will have little opportunity to show its potential being surpass from inception.

The AA, while complex, has to perform in such a way as to facilitate some very basic objectives. It is confronted with an unknown future of unknown magnitude just as any portfolio manager trying to outperform the averages. Yet, equation 9 enables a portfolio manager not only to show management skills but most importantly, to show that the value of these skills can improve with time.

Equation 9 also implies controlled leverage scalability based on generated *alpha*. A portfolio manager could leverage his *alpha* skills to outperform the long term market average while still maintaining control over his return enhancing objectives and trading procedures.

VIII - Sharpe Ratio and Jensen's Alpha

The AA could add a new dimension to the Sharpe ratio (SR). But first, one should start with the definition of the Sharpe ratio which is:

$$SR = \frac{R_F - r_f}{\sigma_m}$$

where (R_F) represents the total return of fund F and therefore could include *alpha*. And since the total fund F return (R_F) could simply be the average market return plus *alpha*, we have (see Figure 1):

$$R_F = R_m + \alpha$$

Therefore, we can rewrite the Sharpe equation as:

$$SR_J = \frac{R_m - r_f + \alpha}{\sigma_m} = \frac{0.10 - 0.035 + 0.05}{0.16} = 0.718 \qquad (10)$$

where a separation can clearly be seen for portfolio management skills. Equation 10 also provides a numerical example with an *alpha* of 0.05 which would raise the expected long term portfolio return to 15% ($R_m + \alpha$) - from equation 7.

Separating *alpha* as a reward component in the Sharpe ratio is a reasonable assumption. Not only will the ratio account for the risk premium $\beta(R_m - r_f)$, it will also account for the excess return over and above the average market return (α) due to the portfolio manager's skills and could also provide a better picture for relative portfolio performance comparisons.

A Jensen adjusted Sharpe ratio (SR_J) could be a more reasonable measure of the reward to risk ratio since it would also quantify the portfolio manager's ability to outperform the markets.

A positive *alpha* would shift the ratio higher by a factor equal to α / σ_m without necessarily increasing the market risk (σ_m). While an *alpha* of zero would return the Sharpe ratio to its usual definition, and since:

$$R_F - R_m = \alpha$$

would further imply that the fund F return equals the average market return (R_m):

$R_F = R_m$

In essence, equation 10 could be considered as a simple reformulation of the Sharpe ratio. Either way, separating the total fund return into the average market return and the excess return over and above the market return gives the ability to measure portfolio manager's skills (*alpha*) more easily.

Since the market's long term average historical Sharpe ratio stands at about 0.406, using *alpha* as in equation 10, we could extract from this reading the fraction due to management skills: α / σ_m. Therefore, by increasing *alpha* one could then obtain a higher market Sharpe ratio due to portfolio management skill alone. The higher the *alpha*, the higher the improvement would be and the higher the adjusted Sharpe ratio would be.

It is worth noting that adjusting the Sharpe ratio to Jensen's finding of a negative *alpha* of -1.1% would reduce the historical Sharpe ratio to: 0.337. Based on these equations, the long term return for the average fund F would have been:
0.10 – 0.011 = 0.089 or 8.9% which is close to historical averages.

IX - Sharpe Ratio and Alpha Accelerator

When applying the Alpha Accelerator to the Sharpe ratio we obtain:

$$SR_\alpha(t) = \frac{R_m - r_f + \alpha\,[\,1 + f_a\,]^{t-1}}{\sigma_m} \qquad (11)$$

The enhanced Sharpe ratio inherits an *alpha* power function, and for a positive *alpha* with a positive accelerator, the Sharpe ratio will increase with time.

Looking over a 20 year period, the Sharpe ratio could rise, for example, to 3.59 (a figure that would make the envy of any portfolio manager) with relatively low values for *alpha* and the *alpha* accelerator.

$$SR_\alpha(t) = \frac{0.10 - 0.035 + 0.05\,[1 + 0.13]^{20-1}}{0.16} = 3.59 \qquad (12)$$

As can be seen, the Sharpe ratio time dependent component added considerably to its historical measure. The Sharpe ratio is increasing in time proportionally to the accelerator fraction α^*: the effective alpha.

$$\frac{\alpha^*}{\sigma_m} = \frac{\alpha\,[\,1 + f_a\,]^{t-1}}{\sigma_m} \qquad (13)$$

The implications are far reaching for a portfolio manager; from an historically static measure of reward to risk ratio, he can now look forward to a more positive outlook where the measure of his skill level will increase in time all by applying procedural trading techniques to enhance his performance.

To achieve a Sharpe ratio as in equation 12 would require an effective *alpha* in the order of 50.9% which exceeds by a wide margin what can be seen from the majority of portfolio managers for long term performance.

$$SR_{\alpha} \times \sigma_m - R_m + r_f \approx \alpha^* \approx 3.59 \times 0.16 - 0.10 + 0.035 = 0.509 \quad (14)$$

The expected portfolio return as defined in equation 4 could simply be reformulated taking into account the *alpha* adjusted Sharpe ratio as follows:

$$\mathbb{E}(R_p) = r_f + SR_{\alpha} \times \sigma_m \quad (4.1)$$

where SR_{α} is defined as in equation 11 and since SR_{α} is now a power function, the expected portfolio return will increase as a power function with time. Equation 4.1 is the cornerstone of this paper; it alone represents a major shift in the understanding of *alpha* and its implication in portfolio management. When compared with the already accepted equation:

$$\mathbb{E}(R_p) = r_f + SR \times \sigma_m$$

only the SR_{α} (*alpha*) term has changed to account for the AA. But this little change makes a major difference in the overall expected portfolio return. A portfolio manager by capitalizing on his/her *alpha* can therefore produce:

$$\mathbb{E}(R_p) = R_m + \alpha^* \quad (4.2)$$

And since α* is the effective alpha as in equation 13:

$$\alpha^* = \alpha\,[\,1 + f_a\,]^{t-1} \quad (4.3)$$

then

$$\mathbb{E}(R_p) = R_m + \alpha\,[\,1 + f_a\,]^{t-1} \quad (4.4)$$

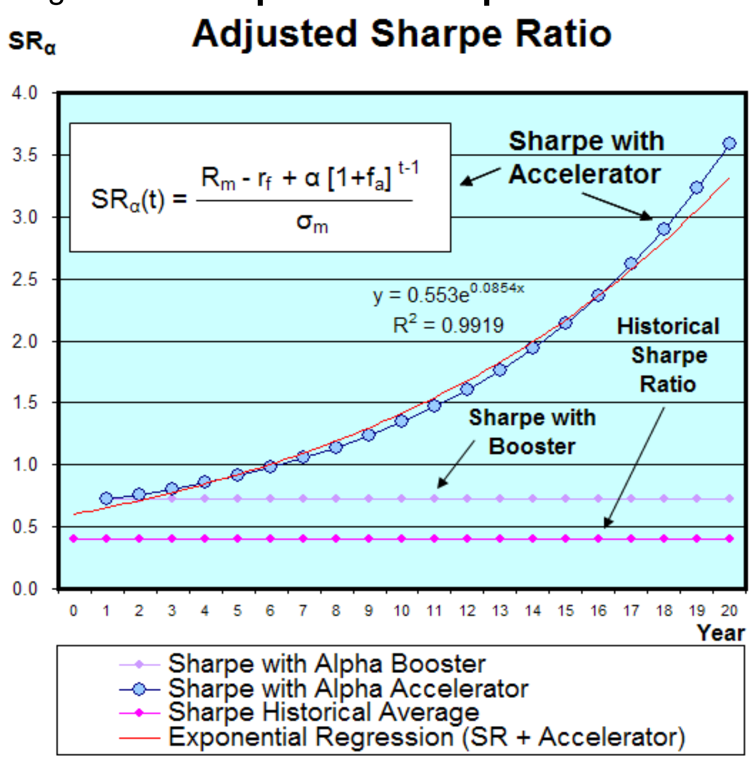
which makes the point very clear that the excess return over and above the average market return is all skill related.

Equation 4.4 was achieved in a totally randomly generated trading environment where you can often find the definition for a zero sum game. Yet, this trading methodology triumphs and grows where it counts the most: in the excess expected return over and above the average market return – meaning it generates an *alpha* power function.

Figure 4 demonstrates that the Sharpe Ratio increases in time when the AA is applied. The starting point is the same as the historical Sharpe ratio, however, as time passes, the ratio increases at the effective AA rate. A Sharpe ratio rising above historical average by the factor as in equation 13 does make obvious that the reward per unit of market risk can increase with time in conjunction with the level of skills. However, a caveat must be injected here. As time (t) increases beyond a critical value (which was not found in current tests), a rising AA adjusted Sharpe ratio should become unsustainable. It will have to reach a point of diminishing return and then decline due to the sheer weight of the portfolio. At present, after hundreds and hundreds of 20 year tests, equation 11 still best fitted the results.

Designing an equation as 4.4 or 11 does not necessarily make it work: they are just theoretical equations. What is required is proof that such equations validate test results and that the best mathematical expression to explain the phenomenon was exactly the equations designed. Applying leverage to management skills is not altogether new; it has been around for more than a while. However, here it is explicitly designed as an incremental procedure which has for only purpose to improve long term return. It is by providing the worst possible trading environment where such equations should be disproved that it is worthwhile to demonstrate the merits of such an endeavor.

Figure 4**: Sharpe Ratio + Alpha Accelerator**



The Sharpe Ratio with the *alpha* accelerator (AA) shows that reward per unit of risk improves with time due to improving management skills.

X - The Implications of Trading Random Price Series

Being able to profit from randomly generated stock price series has always been considered as a trivial pursuit. Nonetheless, throughout this paper claims are made that deterministic trading procedures can not only improve performance over the Buy & Hold but pushes even further and suggests a rising rate of return due to an increasing AA adjusted Sharpe ratio over time. This means that an effective *alpha* (α^*) can improve with time and thereby a portfolio manager can provide more than luck to achieve superior returns.

Imagine designing a trading system where you don't know what the market reserves for you in the nature of price movements: be it in speed, direction or magnitude. Imagine further that all forms of technical indicators lose much of their value in a randomly generated environment and forecasting methods fail to predict future price variations. No fundamental data can come to the rescue due to the very nature of your trading environment. The sum of all trading decisions could only provide results with the most probable outcome close to the Buy & Hold. Aiming for full exposure would promise returns close to the average market upward drift, if you catch my drift (pun intended).

You could statistically slice and dice the data from all past tests, provide all kinds of measures as to what happened in a particular test, and still, most of it would be useless due to the random nature of the data. None of it could help you determine which stock would go up or down tomorrow or which would perform best in your next run; nor where you should put your money or how to distribute it over your selection for best possible outcome.

Now, imagine that you could design a trading system that could obey equation 9 and produce results such that the overall rate of return would increase in time and all the while the AA adjusted Sharpe ratio would also rise over the trading interval. Wouldn't you search all the ways to break it down? See where your reasoning went wrong. And if you could not find any flaw, wouldn't you then try to find new ways to control even better your trading environment with new and improved procedures seeking to either reduce risk or improve return further? Wouldn't you try to gain as much effective *alpha* as possible?

Trading randomly generated price series is not the same as trading randomly. Even though all trades may be triggered as random events being the consequence of random price variations; it can all be done within a deliberate and pre-determined trading plan.

You can pre-determine how you want to trade, what will trigger a trade and under which conditions you are to bail out. You can preset trading conditions years in advance and then wait for those conditions to come true if ever. The buying matrix (of size 50x1000, see Figure 5) will be of the sparse type variety with every positive entry the consequence of a random price variation triggering a preset trading rule. You don't know when a trade will be triggered, only that it will, should the preset conditions be met and this could be at anytime. You would kind of exchange not knowing the when a trade is triggered for at what price it will be according to your preset conditions.

Having given every stock the ability to randomly select the initial quantity to be traded you will have thereby distributed initial portfolio weights at random. At each run, the buy matrix will be different; no two series will be identical. You could not predict which series would have heavier weights nor could you predict the total number of shares that would be bought or when during each run. However, you could know in advance at what price a trade could occur.

Figure 5: **Sparse Buying Matrix**

B0	B1	B2	B3	B4	B5	B6	B7	B8	B9	B10	B11
200	100	200	200	100	100	100	100	100	100	100	100
0	0	0	0	0	0	100	100	0	100	0	0
0	0	0	0	0	0	0	100	0	0	0	0
200	0	200	0	0	0	0	100	0	100	0	0
0	0	0	0	0	0	0	100	0	0	0	0
0	0	200	0	0	0	0	100	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
200	0	0	0	0	0	0	0	0	0	0	0
0	0	200	0	0	0	0	0	0	0	100	0
200	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	100	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
200	0	0	0	0	100	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	100	0
200	0	0	0	0	0	0	100	0	0	0	0
200	0	0	0	0	0	0	100	0	0	0	0
200	0	0	0	0	0	100	0	0	0	0	0
0	0	0	0	0	0	100	0	0	0	0	0
0	0	200	0	0	100	100	0	0	0	0	0
0	0	0	0	0	0	0	100	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	100	0
0	0	200	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	100	0	0	0	0	0	0
0	0	0	0	0	0	100	0	0	0	0	0
0	0	0	0	0	0	0	100	0	0	0	0
0	0	0	0	0	0	100	100	0	0	0	0
0	0	0	0	0	0	0	0	100	0	0	0

The buying matrix will be of the sparse variety. Less than 5% of entries will be different from zero. There is no way of knowing when an entry will be positive, nor how many trades will be taken or the quantity traded.

Procedure after implemented procedure had to show incremental *alpha* and it is only on this basis that procedures were kept. Control functions were designed; boosters and accelerators were added to obtain an overall result where equation 9 could thrive. Holders (no trades), enablers, enhancers and stop loss functions were included to maintain objectives on course. It was the sum of all these implemented procedures that allowed a rising rate of return and a rising AA adjusted Sharpe ratio (see equation 11 and Figure 4). It turned out that a polynomial function correlation was a better fit than the linear correlation for the AA adjusted Sharpe.

Whatever trading technique you use, you want first to generate *alpha*. You need to add *alpha* to the basic Buy & Hold equation as stated below:

$$\mathbb{E}(P_v) = C_o (1 + R_m)^t \qquad (15)$$

which will produce:

$$\mathbb{E}(P_v) = C_o (1 + R_m + \alpha)^t \qquad (7)$$

and then, to go further, by adding an *alpha* booster:

$$\mathbb{E}(P_v) = C_o (1 + R_m + \alpha [1 + f_b])^t \qquad (8)$$

but even better still, adding an *alpha* accelerator:

$$\mathbb{E}(P_v) = C_o (1 + R_m + \alpha [1 + f_a]^{t-1})^t \qquad (9)$$

In all cases, there is no modification made to the average market return (R_m). What you do in the market has too little influence to even be considered as having any impact what so ever on the course of the market as a whole. So your actions all have to be at the expertise level. It is what you do, when you do it that can make a difference. Just being a better stock picker, for instance, could generate positive *alpha* for equation 7, however, in randomly generated data the notion of better stock picking is totally irrelevant (due to randomness) as there is no way of knowing or forecasting which series will outperform the others.

Should you not be able to show any *alpha*, then all 4 equations (15, 7, 8 and 9) are the same and will provide the same shape, the same origin and the same output: namely the Buy & Hold equation.

XI - The Test Results

What has been presented so far is the explanation for the results obtained from hundreds of tests done on randomly generated price series in order to simulate a market. The AA was the equation that best matched the results and served as explanation or mathematical expression that also best described the phenomena under study. Just as the Buy & Hold equation resumes all in the way of performance and end result, the AA described best the over performance over time due to leveraging market expertise and portfolio management skills.

Figure 6 presents the graph of random prices generated during one run. Since no fixed seed was used with the random number generator, each run had totally different result and could not be duplicated. However, the general shape of Figure 6 was somewhat preserved even though no two series could have the same prices in any particular run or successive runs. For the particular run in Figure 6, 10 stocks or 20% of the group failed by dropping below zero.

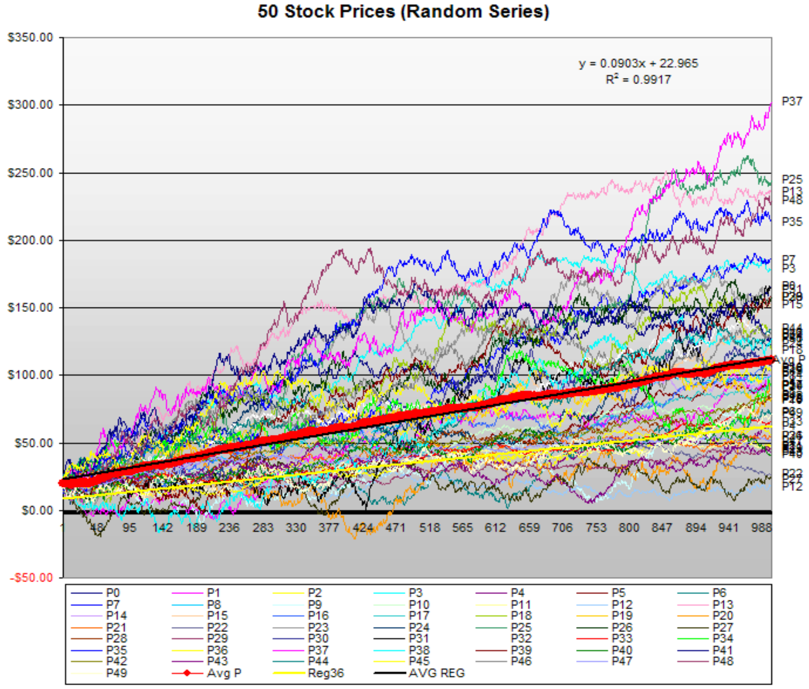
The Buy & Hold strategy, by taking the same initial position in each stock at the beginning of the test and holding till the end of the investment period, would have the same general form as Figure 6 except for the scale (see Figure 7). The highest priced stock would end up with the highest weight in the portfolio as depicted in Figures 6 & 7.

The disadvantage of the Buy & Hold strategy is the “hold” portion of the strategy. What ever happens during the investment period, one waits and waits holding till the end of the period. In the end, the stock that rose the most will have the highest weight in the portfolio and will have contributed the most to the overall portfolio performance while the stocks that have not risen much during the period would have been a drag while the stocks dropping to zero would have generated losses. The average rate of return would have followed the average drift (middle red lines in Figures 6 and 7). The Buy & Hold is not necessarily a bad strategy. You could ask Warren Buffett what is his preferred holding period and he would reply: “Forever”; and he has done pretty well just following that strategy. In the Buy & Hold, you play the whole group of stocks; the best performers will end up with the heaviest weights in the portfolio as a direct consequence of the methodology used.

In Figure 7, the average drift can clearly be seen. Since all prices were randomly generated, the linear regression of the average performance has an R^2 of 0.9977 which is a good indication for the average drift as it should. The advantage of using the linear drift model is that the sum of straight lines will also produce a straight line. The sum of 50 prices series with expected zero mean should also produce an average mean close to zero as well. This can be seen by the high linearity for the average price (middle thick red line).

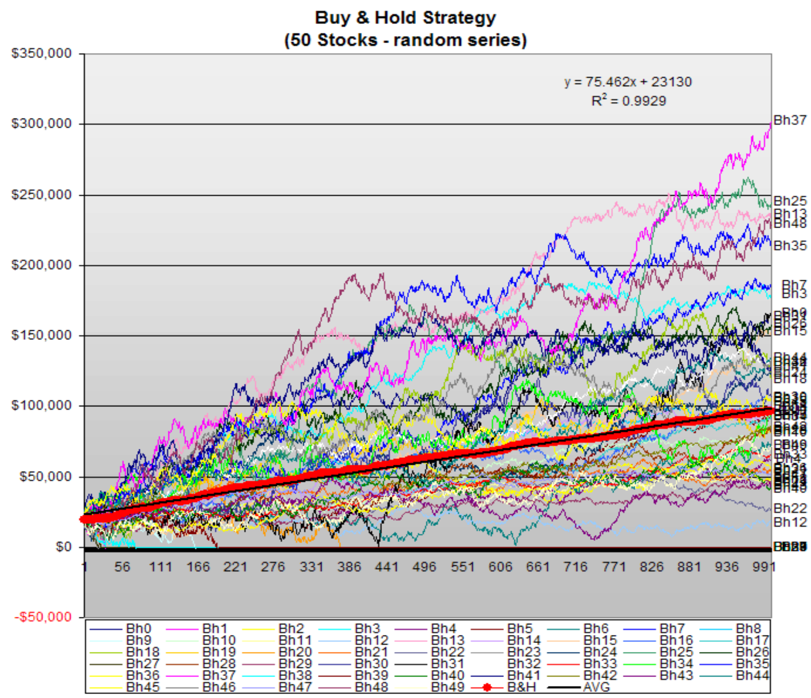
Applying the AA to the same group of stocks where best performers are rewarded by buying more shares while the worst performers are neglected to the point of having their number of shares reduced to zero will provide a graph as in Figure 8 where the highest priced stock might not necessarily be the best performer of the group but could still produce remarkable returns.

Figure 6: **Random Stock Prices**



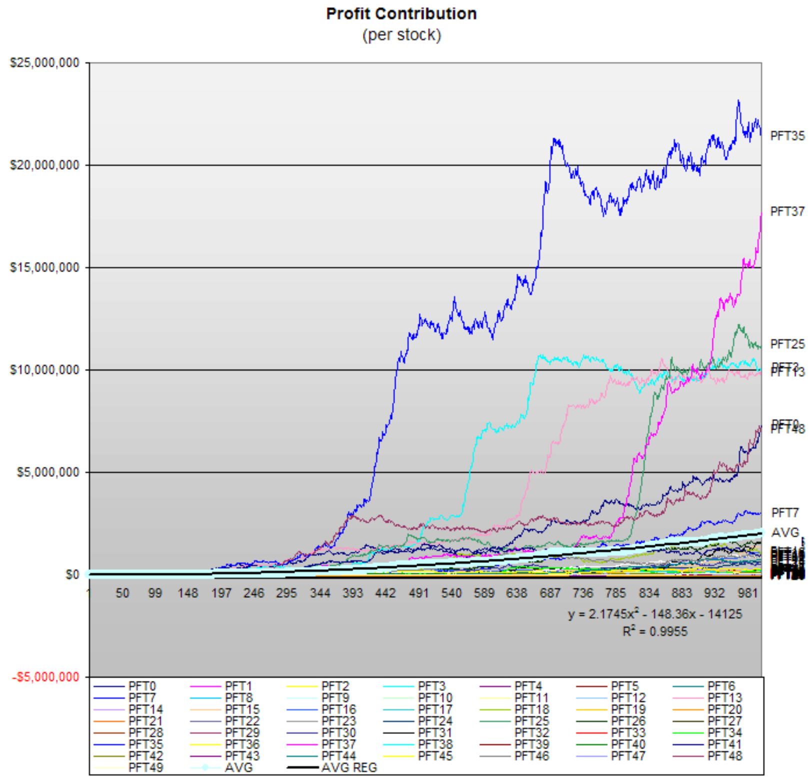
Prices have been randomly generated with about an average $0.09 drift per week. Each run provides a unique set of 50 price series where up to 28% of the group can fail (price dropping to zero).

Figure 7: **The Buy & Hold**



The Buy & Hold strategy should have the same general form as Figure 6. An initial position is taken in all stocks and held till the end of the investment period. Stocks that fail (dropped to zero) lost their stake and were not traded again for the rest of the period.

Figure 8: **AA Stock Profit Contribution**



Applying the AA changes the stock weights in such a way as to put more emphasis on best performers while limiting investment in underperforming stocks.

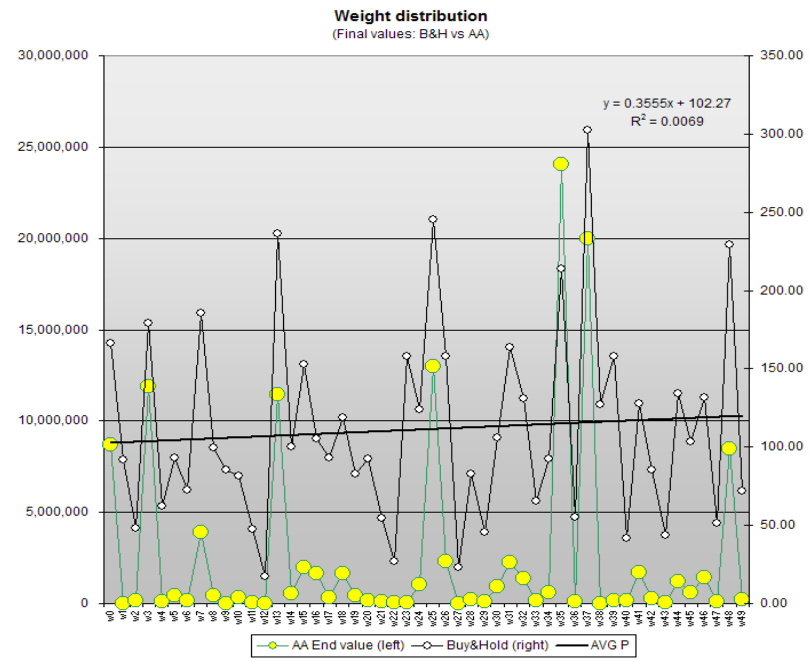
Figure 8 shows the profit contribution by each stock in the group. It can also be seen that the average performance (thick blue line) has for best fit a polynomial equation representing the increasing rate of return over time. In this particular run, stock PFT37 ended up as the highest priced stock in the group, however, it is stock PFT35 that contributed the most to the total portfolio; followed by stocks PFT25, and then PFT25 followed by PFT02, PFT13 and PFT0 (see Figures 8 and 9).

Stock portfolio weights will change with time. Under the AA trading rules, there is no way of knowing which stocks will have the highest weights, however, one can be assured that in the end, the highest weighted stocks will provide the highest portfolio profits. Stock weights will change over the investment period in favor of best performers. The majority of stocks will show performances below average (see below the blue line in Figure 8).

Figure 9 show that some of the highest priced stocks had the highest weights but not all. The weights shift in time as if rebalancing in favor of the highest priced stocks, such that low performing stocks will be neglected while higher performers will grab the heaviest share of the weights. Each run will produce different results.

Figure 10 show the cash requirements over the investment period following the AA strategy. Here again, a polynomial function best describes the behavior. Initially, there is a cash outlay to establish stock positions, then during the early accumulation process, additional funds are put in the market till a maximum is reached, afterward, no new cash will need to be injected in the system. It even goes to a point where the excess cash available is under-utilized (this is where I need to do some more work with the expected result that it will increase the overall return by better use of excess equity).

Figure 9: **AA Portfolio Weight Distribution**

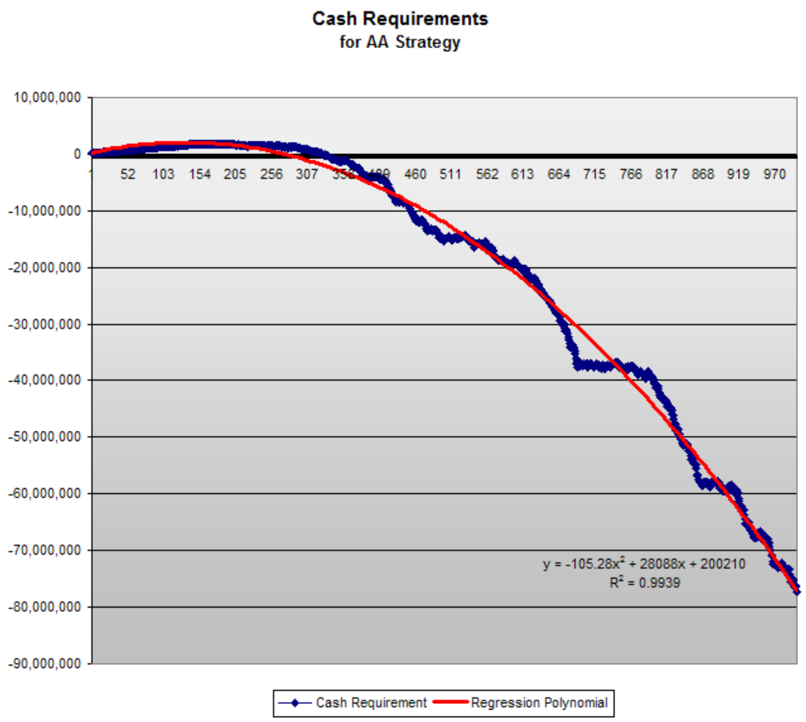


The weight distribution is shifted in favor of best performers. Some of the highest priced stocks (right scale) producing some of the highest returns (left scale). The correlation coefficient R^2 makes the point that the prices series are uncorrelated over the investment period with a low 0.0069 value.

Figure 11 shows the fifty 1000 week linear regressions (19.2 years) for all the stocks in a particular run as well as the average for the group. It can be seen that the average drift is about $0.11 per week or $0.022 per trading day which is close to design specifications. At each run, Figure 11 might seem the same on the surface since the general form would be preserved; however each line would depict a different stock with different results. The sum of the lines divided by 50 provided the average drift for the group (the straight thick blue line in the middle).

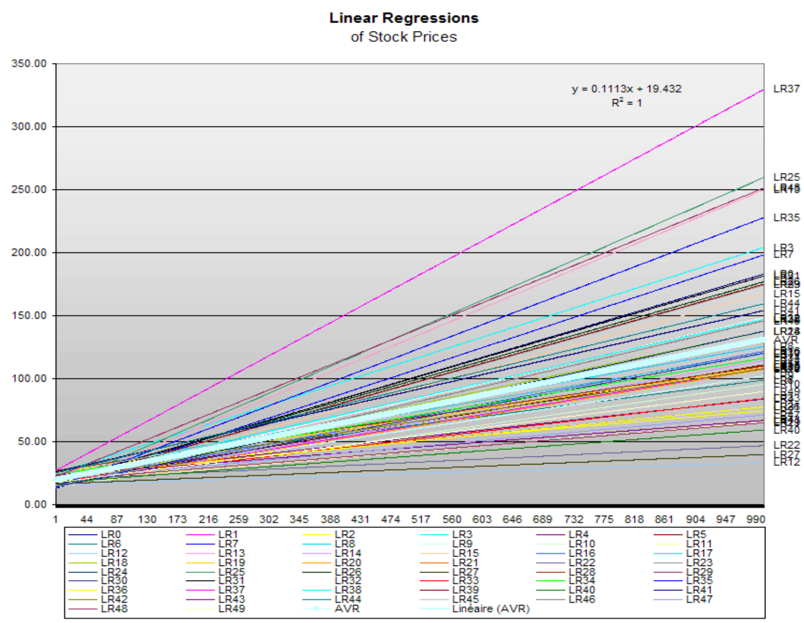
Taking a random selection of 50 stocks from the real stock market and tracing their respective 20 year regression lines on a single chart would provide about the same graph as Figure 11 except for a few which would have a much higher slope.

Figure 10: **AA Cash Requirements**



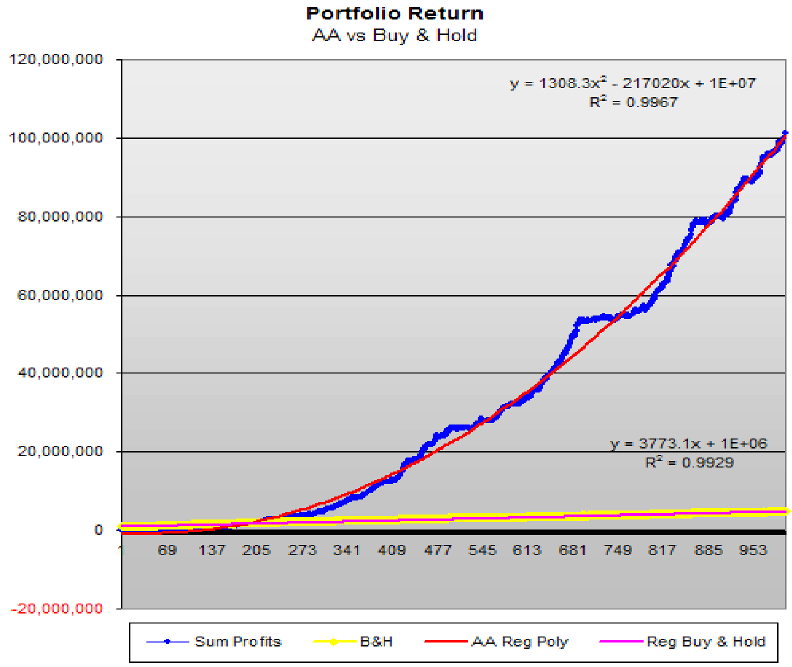
The AA strategy has an interesting behavior. After some time, the average cash required to implement the strategy reduces to a point where the strategy is self sustaining.

Figure 11: **Linear Regressions of Stock Prices**



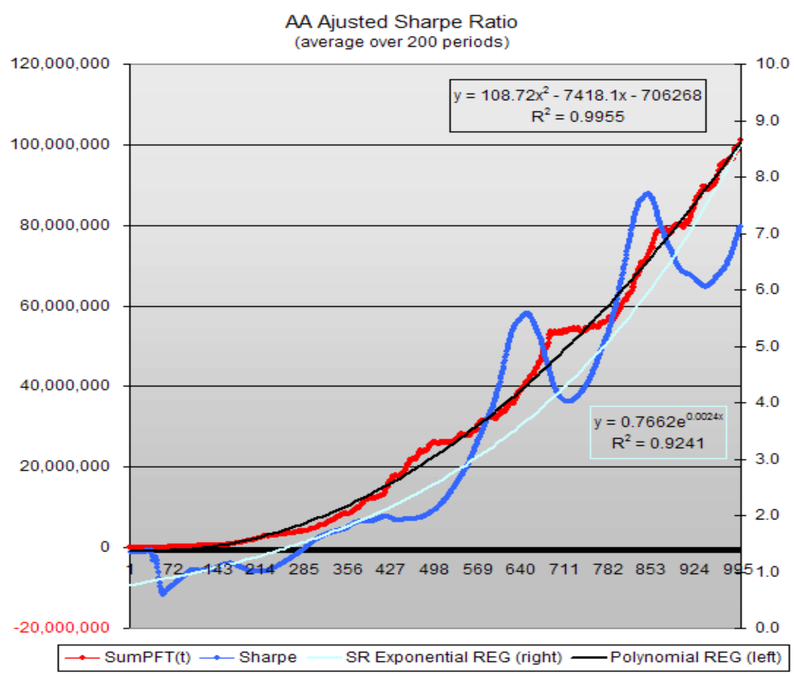
The linear regressions of all stocks in a particular run will show that their sum will also be linear. The correlation coefficient R^2 makes the point clear with a 1.00 value.

Figure 12: **Portfolio Returns (AA vs B&H)**



When comparing an AA portfolio with the Buy & Hold it can be seen that the best fit for the AA is a polynomial function while a linear regression best describes the Buy & Hold. In either case, the correlation coefficient R^2 is sufficiently close to 1.00 to make the point very clear.

Figure 13: **AA Adjusted Sharpe Ratio**



The graph shows total portfolio return (left scale) with the AA adjusted Sharpe ratio (right scale) over the investment period (19.2 years). An exponential function best described the evolution of the Sharpe ratio.

Figure 12 compares the AA portfolio return to the Buy & Hold strategy. What is being presented is the liquidation value for each portfolio at any time *t*. It can be observed that the best regression line for the AA portfolio return is a polynomial of order two which indicates that overall performance is accelerating. The AA portfolio strategy is clearly an over achiever and represents a way of increasing performance way above the Buy & Hold strategy even though by test construction you were not expected to outperform the market average.

Figure 13 shows the AA adjusted Sharpe Ratio. The best fit for the Sharpe ratio being an exponential function as described in preceding sections. To have an adjusted Sharpe ratio that increases in time while playing in a randomly generated environment simply stresses the fact that leveraging portfolio skills (*alpha*) through predetermined trading rules can increase performance (see equation 11).

Figure 13 also helps make the point that equation 11 is a good approximation for the AA trading strategy. It is the delayed *alpha* acceleration fraction of the AA adjusted Sharpe ratio that is the main reason for the rise in time and thereby validates the approach taken as well as the mathematical expression for the phenomena. This again stresses the importance of leveraging expertise in the portfolio by managing the inventory more closely to meet return objectives. This does not say that you have total control over the final obtained results, only that what ever the final results, they would have been optimized through the AA.

Test results also show that the weight of each stock in the portfolio is continuously changing following the vagaries of price movements. However, as time evolves, weights are gradually shifted in favor of the best performers of the group, often making the highest price stock the heaviest weighted stock in the portfolio.

Weight shifting is done in favor of some of the best price performers while the worst performers are starved out. This is like moving away from the efficient portfolio that tries to reside on the efficient frontier, as if to increase return above the historical average you had to move in the opposite direction than the one which led to the efficient portfolio. As a side effect, stocks not rising or dropping to zero would show a relatively small profit or loss respectively while the highest rising stocks would show the greatest profits. Also, another side effect is that a portfolio will have a tendency to show a net profit (net liquidating value) early on and improve further as time evolves (see Figures 12 and 13 as examples). A scenario which could withstand the saying: “Cut your losses and let your profits run”.

This leads to that trying to rebalance on the conventional Sharpe ratio is like shooting oneself in the foot, all in the hope of having what is called an “efficient portfolio”. From equation 10, any modification in the Jensen adjusted Sharpe ratio would most likely and most probably be due to a change in *alpha*. To maintain the Sharpe ratio within a specific range would mean to reduce *alpha* when ever the Sharpe ratio exceeded the admissible range; this in turn, could only be done by selling leaders (reduce weights of best performers) and buying laggers which would increase their weights in the portfolio. In the end, you would be left with small positions in the leaders and huge positions in the laggers with the only advantage of having a relatively stable Sharpe ratio. The side effect would have been to almost assure yourself of having at most what is defined as an "efficient portfolio" with the consequence of achieving at most only an average portfolio return having destroyed your *alpha* in the process. The object of the game is not to have a stable Sharpe ratio; it is to make as much money as you can without undue risk.

The same goes for the philosophy behind an index fund that tries to mimic a particular index. The rebalancing is done as to maintain the relative portfolio weights of the index and here again; this can only be done by selling part of the best performers and using the proceeds to add to laggers all in the hope of achieving R_m: the average market return.

It is not by endeavoring to limit your return by all possible means that it will increase; it will do exact what you are calling for. It all should be looked at in a different light. One should escape this "efficient frontier" and go in the opposite direction, in the direction of an increasing *alpha*.

It's like participating in a special 50 horse (stock) race with each having equal odds to finish the 20 furlongs (years) from the starting line (t_o). You know from such past races that almost a third (15) will not cross the finish line. Of the rest, most (25) will be part of the also ran (no big money) while the remaining runners (10) will grab most of the prize money in finishing order at the finish line. Reasons for horses not finishing the race will be: lame, sick or dropping dead on the track (bankruptcy, fraud, book cooking or self serving interest). For the 35 horses finishing the race, most (25) with get small prizes (low and close to average returns) and will have suffered from being: outclassed (bad business models), no stamina (under capitalized) or poor jockeys (bad management). Of the 10 or so to finish up front, the winner will get the biggest chunk of the prize money (heaviest portfolio weight) followed by the other front runners which will take the biggest part of the remaining prize money and the rest will be divided in decreasing finishing order (according to weights) to the also ran (those that dared finish the race). For best illustration see Figures 6, 8 and 9.

You know from experience that betting on a single horse (your favorite #7) is a gamble where you could loss everything, get out even or win big (odds of 15:50 to lose it all; 25:50 to come out average; 10:50 to make above average and 3 to 4:50 to make it big). However, in this special race, you have a positive expected return should you spread your bet on all runners. And by spreading your bets, the most probable outcome has odds of 1.10:1 compounded per furlong: $(1 + 0.10)^f$.

So, the decision is relatively simple, you place 50 initial bets, but instead of waiting for the race to finish for cashing in your winnings (Bet & Hold), you change your bet size (position sizing) as the race progresses. You shift the money placed on the trailing horses to the front runners and increase their bets (leverage). Throughout the race, weights will be shifted around depending on the evolution and the make-up of the race itself, but no matter, at the end of the race, most of your money will now be distributed in the finishing order of the best performers (see Figures 8 and 9).

Shifting weights around (position sizing) will be regulated by your betting methodology (see equation 16 in section XIII) and will be done within specific and predetermined parameters such that whatever the outcome of the race your final and biggest bets will be with the best horses (winners) in their order of finish. And your smallest bets would have been with the losers (those that did not finish the race). Thereby, winning big on your winners and losing small on your losers.

Having made your initial bets and waited for the race to finish (Bet & Hold), you would have had an expected return of $(1 + 0.10)^{20}$ = 6.7 times your money (Figures 12 and 17). Had you shifted the weights of your wagers in favor of front runners and then applied leverage, the final result could be 190 times or higher your initial capital. This would depend on how much capital you had to deploy and how much leverage you decided to apply as the race evolved (see Figures 12, 17 and specially 18). To put this in perspective, a $ 1 000 000 stake would have increased over the 20 year period to $ 6 700 000 under the Buy & Hold, while using the AA trading technique, it would have increased your initial stake to $ 190 000 000. Adding a zero or two to the initial stake will also add a zero or two to the outcome.

The results were independent of how the race was ran, your method of play made all the difference. And you knew, in advance, that whatever the outcome of a future race, your betting methodology (stock allocation, trading technique and position sizing) would put you way ahead of the pack.

XII - The Trading Environment

Equation 9 is the basis for generating the trading environment. It states that a portfolio manager can leverage his/her management skills in such a way as to increase in time the overall rate of return to a point where it far exceeds the Buy & Hold strategy.

But a trading environment needs more. It needs feasibility, executability and scalability.

Feasibility was hopefully demonstrated throughout this document: equation 9 can serve as the foundation for implementing a family of trading strategies that have for objective to accelerate return by leveraging portfolio management skills.

Executability was more than satisfied since executed trades were in the hundreds of shares at a time. Equation 9 depicts more an accumulative process than an in and out day trading procedure; it is a long term trading plan with intended long term holding periods. The trading strategy never gets into a situation where you have to flip a million shares of low priced stocks every day to make a profit. It counts on its accumulation program to increase the share count gradually as price permits.

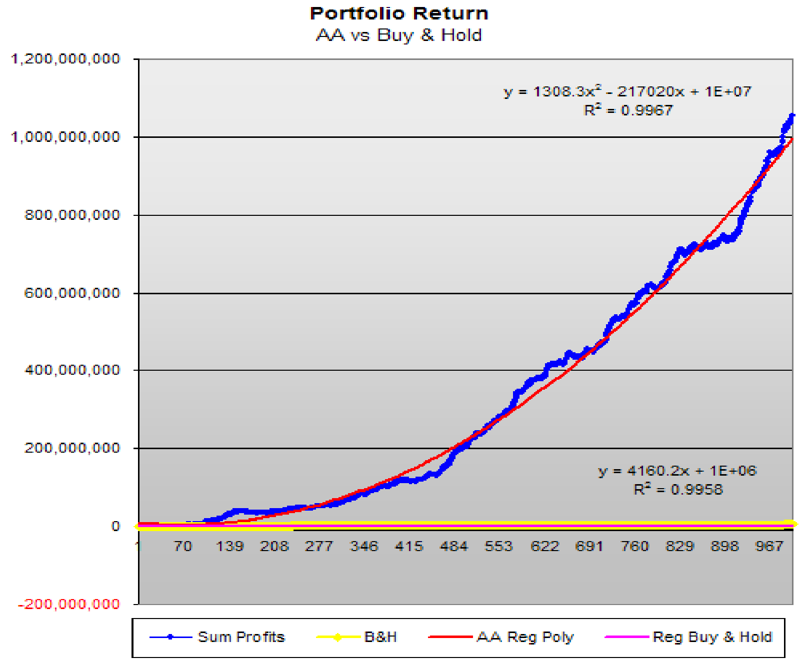
Scalability has not yet been address in this paper; however, the AA trading procedures can be scaled up to a point. You want more return, then leverage skills a little bit more or increase your trading basis (number of shares traded at a time). Wanting higher returns will require more initial capital to implement a higher share accumulation rate. However, as was presented in Figures 10 and 15, the cash requirement tends to taper off after the initial accumulation process and then an over availability of funds occur as the system, in time, becomes self-sufficient.

By being just a little bit more aggressive (see Figure 14 compared to 12) in the accumulation program by allowing a slightly greater number of shares to be bought in each stock will lift overall performance greatly from the same basis with relatively low added capital requirements (see Figure 15). Wishing to push performance still higher only requires added pressure on the excess equity utilization functions (see Figure 18). And since the trading methodology becomes self-sufficient in time, better equity utilization functions could be designed to improve return further. As time advances, a lot of excess equity goes unused.

When applying scaling as in Figures 12, 14 and 18, it can be seen that total return is increased, initial cash requirements are for a shorter duration and each curve maintains the same shapes as in Figures 13 and 10. Figure 15 also states that after the initial cash requirement to establish initial positions has reached a peak, it then passes in negative territory implying that the market is then supplying the required cash to continue the share accumulation program.

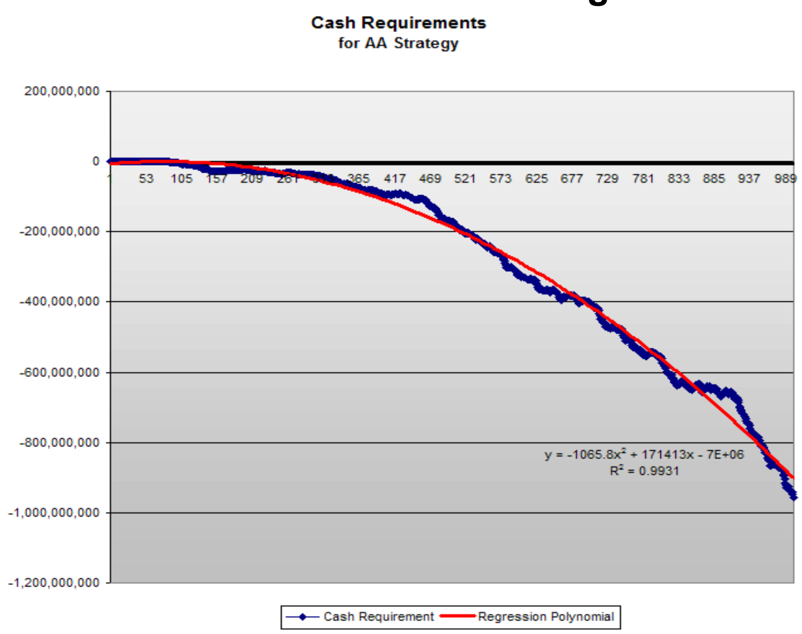
Figure 14 shows that the AA adjusted Sharpe ratio maintains its exponential linear regression line which is in accord with equation 11. Since sooner or later, the AA strategy becomes self-sufficient; one could increase return further by using the available excess equity. In Figures 10 and 15, a negative number stands for cash extracted from the market whereas a positive number represent cash put in the market.

Figure 14: **AA Scalability**



The graph shows total portfolio return (left scale) with the AA adjusted Sharpe ratio (right scale) over the investment period (19.2 years). An exponential function best described the evolution of the Sharpe ratio. The results shown are from a different run than the previous charts, incremental scaling factors have been implemented for this run. See also Figure 12.

Figure 15: **AA Cash Requirement after Low to Moderate Scaling**



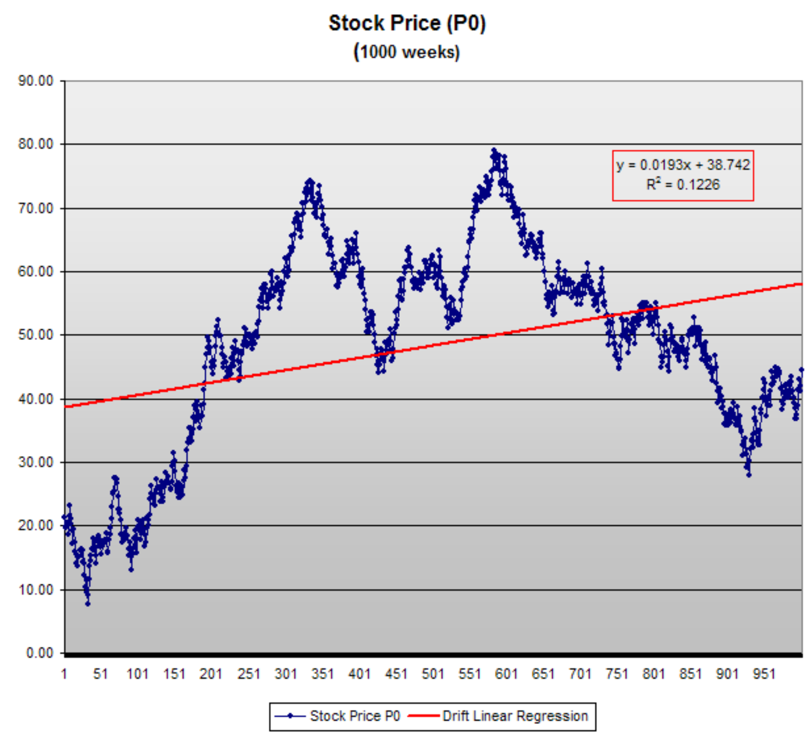
The AA strategy after some low scaling applied. The same typical chart behavior is present: after a certain time, the average cash required to implement the strategy reduces to a point where the strategy is self sufficient. Notice the change in scale compared to Figure 10.

One more aspect of this trading environment relates to feasibility, namely: automated trading. Can an AA trading strategy be implemented and traded automatically? The answer is: yes, definitely. All the trading conditions are preset by the portfolio manager with the advantage of knowing at what price future trades will be executed and this from day one. You might not know when a trade might be executed, but you do know under which conditions it will be, if ever. And this is sufficient to build a trading program that will execute trades automatically.

XIII - Other Considerations

All the tests were done on randomly generated data. The biggest advantage was probably the fact that the random part of the 50 data series were uncorrelated, as they should, being random price series. Also, using 50 price series provided a high probability that the average mean would be close to zero for the sum of the random variations, meaning that a value close to zero would be added to the average drift for the group which explains the high drift linearity (see Figures 6 and 11); this in turn mainly due to the fact that the random components of the price series were uncorrelated. It also represented a high degree of diversification as the whole group of 50 stocks was used in the tests.

Figure 16: **Random Stock Price (P0)**



The graph shows a typical 1000 period randomly generated price series with drift for stock P0. The random component could be considered as white noise. Different chart patterns can be observed on the chart.

Figure 16 shows a typical randomly generated stock price series where the red line represents the drift (linear regression). The random component (noise) overwhelms the signal (drift). The signal to noise ratio is quite low. In this example, the drift is about $0.0193 per period while the noise is counted in dollars.

Removing the drift, one is left with the random component which has an expected zero mean. Technically, there should be no way to profit from such a price series as there is no forecasting method that could predict the future price. There is also no technical indicator that could forecast the next price movement even if, on occasion, a particular indicator could coincide with the move.

The future occurs only once! There is no rerun available. Having a totally random trading environment has the advantage of executing a completely new scenario for each test where duplication is highly improbable. No past scenario could help in designing a trading system except for the group of stocks as a whole. It was looking at the total picture that provided clues as to what to do to profit from randomly generated data. The entire process being under the assumption that there is an average positive drift present in real US markets. And for the last 200 years of US stock market history, there has been no 20 year period that has ever shown a negative return.

It was first necessary to develop a trading philosophy and then a trading methodology that could implement equation 9. Equation 9 is the explanation for the observed phenomena as depicted in Figures 6 through 15 and 18, and not the controlling function.

The AA's controlling function has more the aspect of the equation which follows:

$$H(t) = Q_o \bullet [(1+\Phi) \bullet f(s) \bullet f(e) \bullet (1 + f(\alpha/\alpha_b) \bullet \zeta \bullet (1+r+\alpha+\psi+\Lambda))]^{t-1} \bullet f(x) \bullet P_o \bullet (1 + r + \alpha)^t \quad (16)$$

where r and α have the same meaning as presented in this document. The holding value at time *t* (H(t)) being controlled by all the parameters in the equation separated by those affecting P_o (the initial price) and Q_o (initial quantity bought). All the functions (some not included) are for controlling the behavior of the stock accumulation program. What this equation performs is to preset the trading behavior of the portfolio manager over the investment period. One has no control over how stock prices will evolve in time; however, one has total control over which conditions will enable his participation in the game and from there, execute preset trading rules. Equation 16 illustrates well that the AA is not a price forecasting method but a predetermined inventory controlling function. It becomes how the quantity of shares on hand is being treated that counts: it's the inventory control or the position sizing over the whole set of selected stocks that finally matters. It is also how weights are shifted in time to favor the best performers of the group while at the same time reducing weights in under-performers that helps push performance higher. At its most basic, equation 16 is essentially a Buy & Hold equation with the added twist of selectively and pre-deterministically reinvesting generated *alpha*, or better yet, capitalizing on the portfolio manager’s above average market skills.

Ignoring *alpha* in the controlling function will revert equation 16 to the Buy & Hold equation as it should. Equation 16 is leveraging *alpha* by reinvesting part of the realized *alpha* profits just like reinvesting dividends. The degree to which this *alpha* leveraging can be applied depends on the portfolio manager’s aggressiveness. Without *alpha*, equations 15, 9 and 16 are all the same.

No trading commissions were taken into account in these tests. The reason being very simple: their low impact on overall performance. Trading in quantities of a few hundreds shares at the time and having brokers offering fees of $0.01 per share, total commissions amounted to between 0.004 and 0.008% of final equity. Not something that would make a dent performance wise.

Having all price series generated at random removed some of the volatility present in real markets where trends can last for months. In real markets, price movement correlation can be in the 25% range compared to zero in the random test. The random data had no auto-correlation except of course for the drift. And having used 50 price series where the random components had to have a zero expected mean; their sum would also have to exhibit an average mean of zero as well. This single point would help make the case that real stock prices move in a quasi-random way and are not totally random.

A future point of study will be using random polynomials to represent drift with more random price shocks and forced correlation in an attempt to better simulate a market with pockets of price volatility. But first, testing will go to 100 stocks by 2000 weekly periods to see the impact on the AA adjusted Sharpe ratio as it is expected to start easing and probably start declining a bit somewhere over the 1000 period. Also under research is a quest for an accelerator to the AA, ways to extract even more as time progresses by a better use of excess equity.

XIV - In Real Life

How does all this apply to real life stock trading? For one, in real life, the future also happens only once, and you only have one shot at it; there are no reruns so you better make it your best shot possible. What ever will be will be.

What was presented in previous sections, trading using the AA as in equation 9 or 16, did not say that you could win once in a while; it said that under that specific random trading environment (which tried to closely simulate the real world) you could win all the time, for every single test run done, no exception.

This even if the game presented was a zero sum game where all you could do was hope to achieve at most an average performance. You still managed to outperform the averages by what could be considered a wide margin on every single test. I do not know how to put more emphasis on this point: every single test outperformed the Buy & Hold strategy, and under certain predetermined selected conditions, by a wide margin.

The random environment was constructed according to strict specifications:

1) a set of 50 stocks chosen at random, (random prices: P_{oj})
2) each stock having a random component, where the cumulative sum of all variations ($\sum \varepsilon_j$) would approach zero,
3) each stock having a random linear long term drift, (trend a_j)
4) the group of stocks having an average positive drift, ($(\sum a_j)/n > 0$)
5) price variations having random shocks (Paretian distribution).

In real life, all of these conditions are met with only slight variations. In the real market, stock prices exhibit quasi-random price movements (items 2 and 5); there is a long term secular trend, be it up or down (items 3 and 4); the linear regression of any stock will represent its drift (item 3); prices can have major gaps up or down (item 5) and one could easily choose at random 50 stocks to trade with (item 1).

There is no reason to leave the first item of the list to randomness. One could choose stocks based on fundamental data, pricing models or long term stock ratings provided by services such as Zack's or Moody's. Limiting the selected stocks to the highest ratings would improve on *alpha* by better stock picking. Improving *alpha* remains the name of the game. Equations 9 and 16 apply for what ever stock is picked. This does not say that such picks will outperform a random average selection, only that you are putting the odds in your favor.

Additionally, one could gain 3 *alpha* points above the average market return simply by eliminating the survivorship bias. The procedure can be very simple: consider only stocks trading above $20 in your selection and liquidate any stock that goes under $15; thereby bypassing any stocks going into bankruptcy by first falling to $10, then to $5, then to $1 and finally to zero. For the stocks operating with an excess return higher than the 3 *alpha* points, consider using some form of leverage to boost performance higher. By boosting *alpha* (see equation 9), you are not boosting the average market return (R_m); you are, however, increasing your expected portfolio value $\mathbf{E}(P_v)$ by pushing your *alpha* higher which is your prime objective.

Other procedures could be implemented to gain added *alpha* and further boost the expected portfolio value higher for those over-performing stocks.

In real life, the portfolio overall volatility would be higher due to the higher correlation with the average. It's no reason to stop playing the game. In essence, you are not playing volatility; you are playing the long term upward drift. And that is what really matters. Also, in real life, some stocks are real high performers generating high returns, way above the restrictions placed on this random test environment. In such cases, the AA methodology would simply soar since it ends with the heaviest portfolio weight in the highest priced stock.

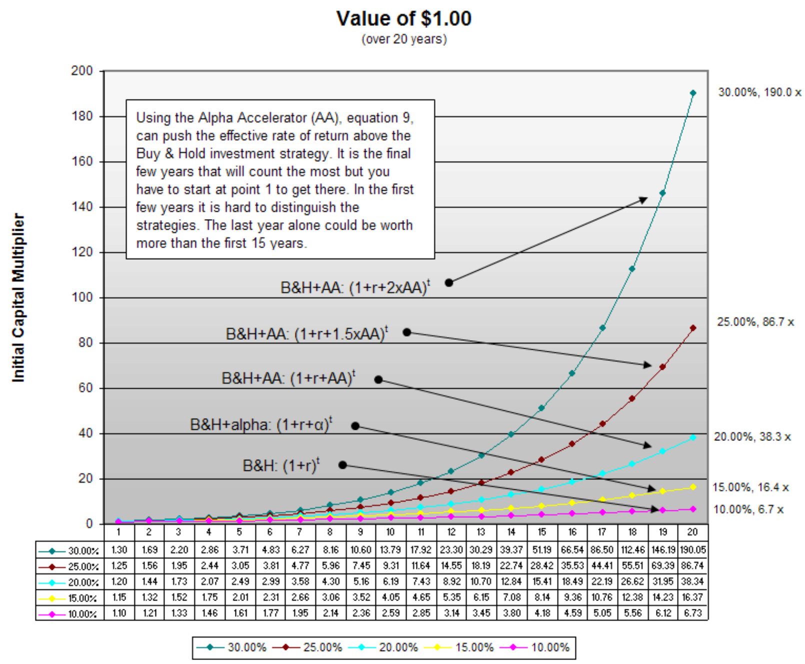
Figure 17 shows for various effective *alphas* the AA equivalent rate of excess return. The major impact is not on the first few years but the last few. To make the point clear, to get there, you had to start at the beginning. By pre-selecting the degree of trading aggressiveness one can set his/her predisposition for an effective AA. It does not guarantee that you will achieve this level of market return only that should future prices maintain an upward drift you will at least surpass what could have been available using the Buy & Hold strategy. To determine the value of the chosen strategy, simply multiply your initial capital by the number appearing next to the rate of return label. For instance, the highest rate in the chart would multiply your capital by 190 compared to 6.7 for the Buy & Hold.

Looking again at Figure 17, it should be noted that the differences will not show up in the beginning as all strategies start from the same origin; it is only with time that their differences will really be visible. But to get there, you had to start, and your real objective is to get to year 20 (finish the race) since that year alone can produce as much as the first 15 years. Pushing the AA higher will have for result higher capital multiplier numbers for Figure 17 or better yet, compare Figures 12, 14 and 18.

It's like selecting at the beginning of the game how you will deal with all price variations to come for all the stocks in your selection; setting your trading plan for years in advance according to the expectation that the market's secular uptrend will be maintained. You are not asking for the market to do more than it did for the past 200 years only that it (the trend) continues; that it does what it has done in the past. You are asking for that 20 years from now, average stock prices will be higher than today; that in 20 years the DOW will be above what it is today.

By presetting your parameters in equation 16, you will be able to extract more according to your degree of aggressiveness which can be regulated by equation 9. It then becomes a matter of selecting your effective rate of return which your preset trading decisions will endeavor to realize in time.

Figure 17**: Future Value of $1.00**



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
30.00%	1.30	1.69	2.20	2.86	3.71	4.83	6.27	8.16	10.60	13.79	17.92	23.30	30.29	39.37	51.19	66.54	86.50	112.46	146.19	190.05
25.00%	1.25	1.56	1.95	2.44	3.05	3.81	4.77	5.96	7.45	9.31	11.64	14.55	18.19	22.74	28.42	35.53	44.41	55.51	69.39	86.74
20.00%	1.20	1.44	1.73	2.07	2.49	2.99	3.58	4.30	5.16	6.19	7.43	8.92	10.70	12.84	15.41	18.49	22.19	26.62	31.95	38.34
15.00%	1.15	1.32	1.52	1.75	2.01	2.31	2.66	3.06	3.52	4.05	4.65	5.35	6.15	7.08	8.14	9.36	10.76	12.38	14.23	16.37
10.00%	1.10	1.21	1.33	1.46	1.61	1.77	1.95	2.14	2.36	2.59	2.85	3.14	3.45	3.80	4.18	4.59	5.05	5.56	6.12	6.73

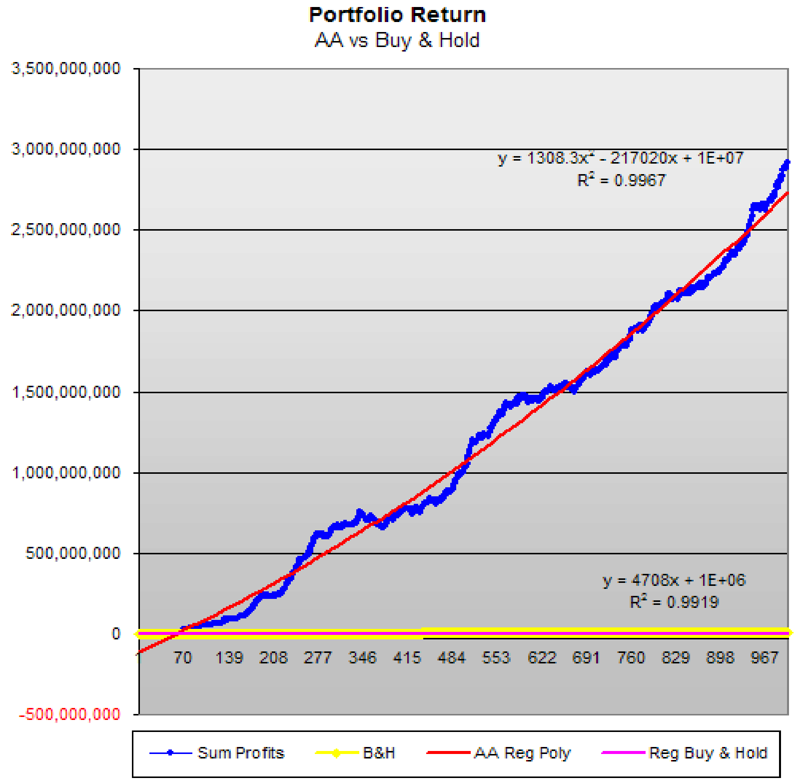
The graph shows the value of $1.00 invested in different trading strategies. From the Buy & Hold to there levels of AA. Whatever the initial capital, simply multiply it with the number in the table at the desire level.

To achieve more than what is presented in Figure 17, it is sufficient to preset a higher degree of *alpha* leveraging. And then follow the preset plan and execute trades as required by the program. This may result in Figure 18.

No use of margin was applied in any of the tests. Using margin, even at quite low levels, would have pushed the overall performance much higher. Only partial realized excess equity was used in all the tests on the same principle as reinvesting dividends.

I think there are three major reasons for this concept (AA) to work. First, the game has a positive expectancy: a long term average positive drift. Second, it is a multi-period game where you can easily change your wager at minimal cost as you go along. It is with this ability to change the bet size (position sizing) that you can put emphasis on optimizing the whole portfolio by rebalancing weights in favor of an increasing *alpha*, and not the other way around, that permits this methodology to outperform. Third, you are playing the averages, you are playing the whole group; you know in advance that some of your bets will fail with relatively small losses (not all your bets will fail however), while most will provide average to above average performance and only a few will thrive with big wins (see Figure 8), you just don't know which ones will be which, but it does not matter. You simply follow your preset trading plan which will take care of the details and sort it all out.

Figure 18: **Pushing for Higher Returns**



Requiring more from equation 16 can push performance to higher levels. It is sufficient to preset the parameters to the degree of aggressiveness that corresponds to your risk taking profile. Compare to Figure 12 or 14.

There are similarities of use of equation 9 in the real world on a macro economic scale. For instance, an accreditive acquisition purchased with a company's accumulated profits or reserves could be considered as a simple reinvestment of excess equity in order to boost performance higher. However, even though profitable, the process is not continuous as presented in equation 9, where only differential action is taken in small increments. Nonetheless, equation 9, just as an acquisition or a leveraged buy-out process, attempts to answer a real question: "What can we do with the accumulated profits"? In the Buy & Hold strategy, nothing is done; while using the AA, part of this excess equity is simple recycled and recycled. Anytime someone uses part of their excess equity to invest in additional holdings, they are in effect leveraging their positions (as in equation 9 by holding more and hopefully for higher returns. It is not an optimal-f problem, a fixed ratio of equity problem or trying to find the optimal Kelly number that is presented in this paper: equation 9 has nothing to do with these concepts. It is by following equation 9, I think, with its own set of limitations that can help you push your portfolio return to higher levels.

Equation 9 has unique and far reaching ramifications. It should change the way we look at stock investing, help modify our perception of the venerable Buy &

Hold investing philosophy; transform it in a new and improved version where "hold" no longer means just wait, but says instead, hold on and accumulate if justified to do so. It goes far beyond a simple modification to a basic tenet of investing; it alludes that its use will result in a rising adjusted Sharpe ratio over time (see equation 11) where most of the added value, if not all, will be a result of a controlled *alpha* recycling process.

Equations 9 and 11 hold new promises for a portfolio manager; they stress the importance of *alpha* and the methods used to enhance it. You might not be able to change the market, but you can surely change your trading behavior in such a way as to implement and extract a much higher expected return than the simple Buy & Hold. Equation 11 also makes the point: that any Sharpe ratio increase is not due to your attempts at changing the market conditions (σ_m , R_m , or R_f) but due to your change in behavior and attitude towards the market. The trading methods adopted, aimed at recycling *alpha*, can be regulated by equation 16 and are the main reason for the increasing Sharpe ratio in time.

XV - Conclusion

This is the first time that such a concept (AA) is being presented (to my knowledge) where one can achieve and sustain a rising adjusted Sharpe ratio over an investment period. By applying an AA to traditional trading methods, one can increase performance by increasing, in time, the reward to risk ratio. It is within the deterministic trading procedures that one can achieve not only to generate some *alpha* but also to reap the rewards of an increasing performance attributable mostly to portfolio management skills.

Although tests were not done for durations longer than 1000 weekly periods, one can still deduce from simple common sense that equation 9 is incomplete as it needs a decaying function that should kick in due to the law of diminishing return. Equation 9 should be considered unsustainable for some *t* value higher than 20 years since for values lower than 20 the power function was still the best fit. It is not that equation 9 breaks down, only that an additional function should kick in to slow down the rate of ascent while still keeping a high rate of return. The next step to research will be to find out at what level beyond the first 20 years that the decaying function could kick in and by how much. For now, one could simply limit himself to the first 20 years and still reap the rewards. I have 20 more years to find the solution.

The methodology used could be described as a glorified Buy & Hold strategy; it wants to change the Buy & Hold for a Buy, Re-evaluate & Accumulate, and then let the market pay for it all. By re-investing part of the profits, just as re-investing dividends, one can generate positive *alpha* and accelerate return. All that is really required is a market long term average positive drift. And as was said before, over the past 200 years there has been no 20 year period with a negative drift in the US stock market.

In essence, this paper says ignore the "efficient frontier", escape and go beyond.

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