Basic Portfolio Math

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Basic Portfolio Math makes the case that certain stock portfolios can tell a lot about their future long-term outcomes based on their past simulated trading behavior.

It could help "predict" within a few percentage points their future value, even some 10 years hence and more.

This goes against many caveats we see about not knowing the future of an automated or discretionary trading system since its past is supposedly no guarantee of its future. True, but still, you could get pretty close to your forecasted expectations. Being able to make such an estimate or forecast is already a plus.

The outcome of a trading strategy, no matter how complex it is, can be expressed as a simple payoff matrix: $F(t) = F_0 + \sum_1^N (\mathbf{H} \cdot \Delta\mathbf{P})$ where the total reward $F(t)$ at time t equals the capital you started with F_0 to which is added the cumulative sum of all profits and losses generated by each one of those N trade.

We have the ongoing number of shares held in the holding matrix $\mathbf{H}$, the price matrix $\mathbf{P}$ which includes all traded prices and more, and the price difference matrix $\Delta\mathbf{P}$. All same-sized matrices. They can grow day by day, one row or one trading period at a time. $(\mathbf{H} \cdot \Delta\mathbf{P})$ is a simple element-wise multiplication of quantity times the change in price from period to period: $(q_{d,\,j} \times \Delta p_{d,\,j})$.

The payoff matrix can be viewed as a concise and elegant representation of any stock trading strategy.

Having an equation to represent a portfolio's outcome, we are forced to consider that whatever we do trading will end up bounded by that equation. After all, we do have an equal sign on the table.

There are not that many components to this payoff matrix either: an evolving inventory matrix and a period-to-period price difference matrix. The holding matrix is composed by adding the sparse **B**uy and **S**ell matrices: $\mathbf{H} = \mathbf{B} - \mathbf{S}$.

You say: *it is more complicated than that.* Well, not really. But, you can make it more complicated if you want, nobody will stop you.

The total profit from any trading strategy is and will be: $\sum_1^N(\mathbf{H} \cdot \Delta\mathbf{P})$. As such, the payoff matrix format makes it a convenient package for analyzing portfolios.

The Long-Term Outcome

On average, the outcome of a long-term portfolio will tend toward the long-term market average, as in:

$$F(t) = F_0 + \sum_1^N(\mathbf{H} \cdot \Delta\mathbf{P}) \to F_0 \cdot (1 + \bar{r}_m)^t \tag{1}$$

where $\bar{r}_m$ is the market's average compounding rate of return.

Your objective is to outperform the long-term market average.

However, you are not the one with the ability to move $\bar{r}_m$, it is out of your control.

Therefore, the payoff matrix is of little help in making you outperform market averages. This is easy to understand, you do not have control over $\Delta\mathbf{P}$ either. Stock prices will change whether you participate or not. All you can attempt to control is the holding matrix $\mathbf{H}$ using your buying and selling procedures.

For a single trade i, we have: $q_i \cdot \Delta_i p_i = x_i$, where x_i is the generated profit or loss on that trade. We can say the total number of trades executed N generated X in profits: $X = \sum_1^N(q_i \cdot \Delta_i p_i)$.

One expression represents a vector of length N while the other is a matrix of size (d, j) with the same N trades. Each row might be the time interval measured in days and each column a particular stock in the portfolio.

For instance, a payoff matrix of size $(5040, 100)$ would provide the total trading history up to the penny of a portfolio holding 100 stocks over a 20-year period (252 trading days by 20 years).

The payoff matrix equation appears complicated when in fact it is very simple. Here is another interpretation: $\sum_1^N(\mathbf{H} \cdot \Delta\mathbf{P}) = N \cdot \bar{x}$. We end up with N executed trades with an average profit per trade of $\bar{x}$, and as such, $\bar{x} = \frac{\sum_1^N x_i}{N}$.

No matter what your trading strategy does, that will be the outcome. And it is simple math.

The Tale of Two Numbers

Any combination of N and $\bar{x}$ will be the total outcome, two numbers where one is just a counter. There is no *secret anything* in that, not even some expertise. It says it all, and yet, says absolutely nothing about how those numbers came to be. N and $\bar{x}$ could be the outcome of absolutely any trading strategy. As a matter of fact, any trading strategy will result in those two numbers since the following equation also holds true: $\frac{\sum_1^N(\mathbf{H}\cdot\Delta\mathbf{P})}{N} = \bar{x}$. This will hold for the smallest to the largest portfolios out there.

Based on the payoff matrix equation, whatever you do, you will be left with those two numbers. That is not complicated. Any combination of those two numbers will be the final result: $X = N \cdot \bar{x}$.

If you want to make $10 million in one trade, then guess what your average profit per trade needs to be and then ask some questions: is it reasonable, is it doable, how much time and how much capital would be required to make it happen? You say: *I will do it slowly, like doing 100 trades per year over the next 20 years*. OK, say you have for expectation: $\$10,000,000 \div 2,000 = \bar{x}$.

Therefore, you would need, on average, a $5,000 profit per trade. That is more reasonable. At least, you solved how long it will take.

But still, a few questions remain: how much capital would be required? What kind of trading strategy would enable you to achieve your goal?

The considerations are: you need 20 years to achieve your goal, you need an undetermined starting capital to do the job, and you need a trading strategy, whether it be discretionary or automated.

The trading methods used do not matter to the portfolio equation since it is only summarizing the results: 100 trades a year is about 2 trades per week.

So, every week for 20 years, based on those 2 trades you need, on average, to make about $10,000. Any trade you lose, you will have to make it back and more to compensate for the lost opportunity. Your strategy's hit rate will become a factor, and trading probabilities in the face of uncertainty will enter the picture.

Beating Expected Market Averages?

You know that the stock market over the short-term is on no one's schedule and that it does not behave according to your desires. It just meanders on its own with no regard for your views, opinions, or trades. So, how will you beat market averages?

You will have to bring in some *skills*. That too can be part of the portfolio formula: $F(t) = F_0 \cdot (1 + \bar{r}_m + \alpha)^t$. The alpha ($\alpha$) is an expression of the added return you bring to the game. And to achieve this, you will have to improve on the trading strategy by providing either a better stock selection, better trade timing, better trade mechanics, better stock allocation, modulated leveraging, or a combination of these.

Should you not be able to generate some positive alpha, you do not need to play the game. Hoping that somehow the market will be especially kind to you and give you the alpha for free might be utopian. A zero-alpha would give you: $F(t) = F_0 \cdot (1+\bar{r}_m)^t$ which, as already stated, tends to the long-term market average. Might as well buy a low-cost index fund and let it do the job for you.

The last formula is actually the expression for an index fund and therefore you should not be surprised if it will be what you will get. But at least, your portfolio over the long term will tend to get close to $\bar{r}_m$, just like some 75$\%$ of professional money managers getting $\bar{r}_m$ or less.

If you try it on your own and get a negative alpha, this would give: $F(t) = F_0 \cdot (1 + \bar{r}_m - \alpha)^t$. The same as shooting yourself in the foot. If you do not know that you will generate positive alpha, why not study a little more to make sure you will get it? Otherwise, buy an index fund.

Over the long term, that positive alpha can make quite a difference. Put some numbers in the equation and see: $F(30) = F_0 \cdot (1 + \bar{r}_m + 0.10)^{30}$ or $F(30) = F_0 \cdot (1 + \bar{r}_m + 0.20)^{30}$. Note that the first expression was achieved by Mr. Buffett over his 50$^+$ year career, and RenTec's Medallion Fund exceeded the second.

What Was And What Will Be

We could divide a trading strategy into two parts. One for its past up to the n^{th} trade, and one for the period from n up to N:

$$\sum_1^n (\mathbf{H} \cdot \Delta\mathbf{P})_n + \sum_n^N (\mathbf{H} \cdot \Delta\mathbf{P})_{N-n} = N \cdot \bar{x}$$

The strategy $\mathbf{H}$ would be the same, meaning it would operate in the same manner as it did in a simulation.

Whatever trading rules that applied in part 1 would operate the same way in part 2. This is understandable since all that was done was separate the time interval in two. Definitely, and most certainly, the price matrix $\mathbf{P}$ will be different as would be $\Delta\mathbf{P}$.

How are your trading rules from part 1 to behave in part 2 under a different set of 100 price series each having its own paths and price variations going forward?

Part 1 was relatively easy to determine. A simulation would do the job and provide the answer since the equation for the total outcome would remain the same: $X = X_n + X_{N-n}$.

Part 2 is the unknown: $X - X_n = X_{N-n}$. If, and only if: $X_{N-n} > 0$, could you make a profit going forward? Part 2 offers no guarantees, you are looking at the strategy's future after all. You can make estimate after estimate, you are still in unknown territory and are left with the notion that your trading strategy $\mathbf{H}$ should behave in the same fashion as it did in part 1.

Whatever was done to get you to n might not strictly apply to get you from n to N.

As n increases in the thousands, you will get a better appreciation for the average trade x_i since it will tend to $\bar{x}_n$ simply as a result of the Law of large numbers (see article: **A Trading Strategy Of Interest**).

Your estimate of $\hat{x}_n$ will get better and better while approaching n: $\hat{x}_i \to \bar{x}_n$. And this estimate of $\hat{x}_n$ could serve as a proxy for part 2: $\sum_n^N (\mathbf{H} \cdot \Delta\mathbf{P})_{N-n} = (N-n) \cdot \hat{x}_n$.

We started knowing nothing about part 2 and here we are with an estimate of where we are going. Say your system made 10,000 trades during its first 10 years (about 4 trades per day), with an average profit of $\bar{x}_n = \$200$ per trade. This would result in an overall profit of about $2 Mil. Based on this part 1, your strategy should do about the same in part 2 and should generate another $2 Mil over the next 10 years.

You did not need to know what the market would do, only what your trading strategy would do, and that is to do the same things as before producing something close to its general trade behavior. If the strategy continues to operate as it did in the past, it should generate another 10,000 trades from year 10 to year 20.

Another article of mine pointed in the same direction: **Use QQQ - Make the Money and Keep IT**.

First, the estimated number of trades $\hat{n}$ was pretty easy to estimate since the strategy was continuously rebalancing 100 stocks every week. The portrayed strategy was expected to do ($12.24 \times 52 \times 100 = 63,648$) trades during its 12.24 years of weekly rebalancing.

The strategy was presented with different initial settings (without changing any of the code procedures) which resulted in different outcomes. Still, the estimate for $\hat{N} - n$ with the added 8 years should come close to ($20.0 \times 52 \times 100 = 104,000$) trades over the 20-year period.

This is not by choice. Well, in reality, it is since the weekly rebalancing trading decision was made, and it is all the strategy did. The rebalancing was the strategy's response to the randomness we find in stock price variations, especially on short-term time intervals such as weekly. Nonetheless, the strategy captured enough to get close to its buy $\&$ hold equivalent. In effect, the strategy played on its weekly variance which over time tends to some constant.

It was by adding features and changing initial trading conditions that you could easily exceed the average market performance which anybody could do without much effort (refer to the cited article for details).

The funny thing about that strategy, even though it made over 60,000 in its first 12.24 years, it would continue at the same pace due to its weekly rebalancing. So, you had the stock selection going for you (having chosen QQQ’s 100 stocks) and earned you profits on the weekly variance of those stock prices where most of the price variations were just random-like with a slight long-term upward bias. This should make you win the game without even really trying, or putting very little skills into it.