A Changing Game

Abstract

The notion is brought forward that even trading randomly over randomly generated prices made to mimic real stock price variations can produce alpha. And from there, designing profitable long-term trading strategies might only require a long-term view of the investment game. Starting with the payoff matrix: Σ(**H**.*Δ**P**), it will be shown that the trading strategy **H** is the most critical part in producing over-performance. And from randomly trading to implementing accumulative trading strategies, there was only a small step to take.

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Other publications:

[1] Alpha Power: Adding More Alpha to Portfolio Return.
[2] Jensen Modified Sharpe Ratio.
[3] The Trading Game.

Website: **http://alphapowertrading.com/**

Excel file used in this publication no longer available.

Randomly Trading

Recently, to answer a question in a LinkedIn forum on the construction of a payoff matrix. I built an Excel file to illustrate such a matrix construction: Σ(**H**.*Δ**P**). Resulting in a 10 stocks by 250 trading days matrix (which could be extended to any size). Price variations were randomly generated. To have a quasi-automated self-contained system, I added randomly generated trades (long and short).

I had trading procedures as well as stock prices being randomly generated. Simply pressed F9 would generate totally new scenarios; same as picking 10 stocks at random from an infinite stock universe. Naturally, the expectancy of such a system, as should be expected, is zero. This was like using heads or tails to play heads or tails which as anyone knows comes out to be the same as playing heads or tails.

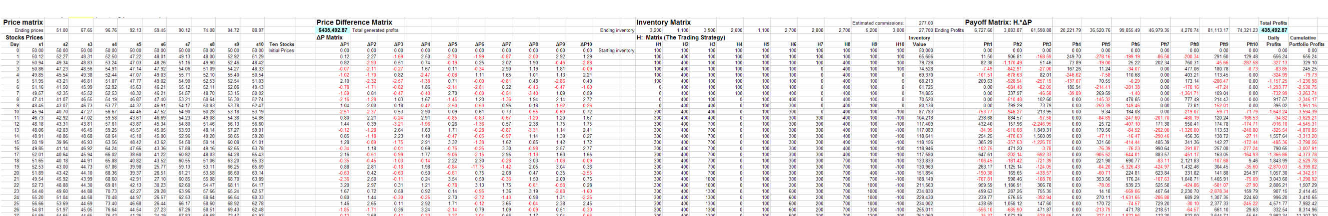
This Excel file (model 1) still illustrated the problems a short-term trader has to face: the unknown and uncertainty. See my other research note on the problems faced with random trading. That particular note illustrates quite well one of the illusions of random trading.

To the original Excel file, I added a low drift and a random gap component in order to have a long-term upward bias and outliers (fat tails) in the price series. Doing so, even if every price variation would remain random occurrences, a randomly generated trading strategy could still be designed to win. At current settings pressing F9 shows a positive payoff Σ(**H**.*Δ**P**) most of the time (over 98%). Meaning that it wins almost all the time for each of the randomly generated portfolios. Not every stock in a portfolio will produce positive results, as would be expected, but for the portfolio as a whole, the result would be positive.

Consider the concept for a moment: randomly generated trades (long and short) over randomly generated price series with low drift and “fat tails”; and you win the game.

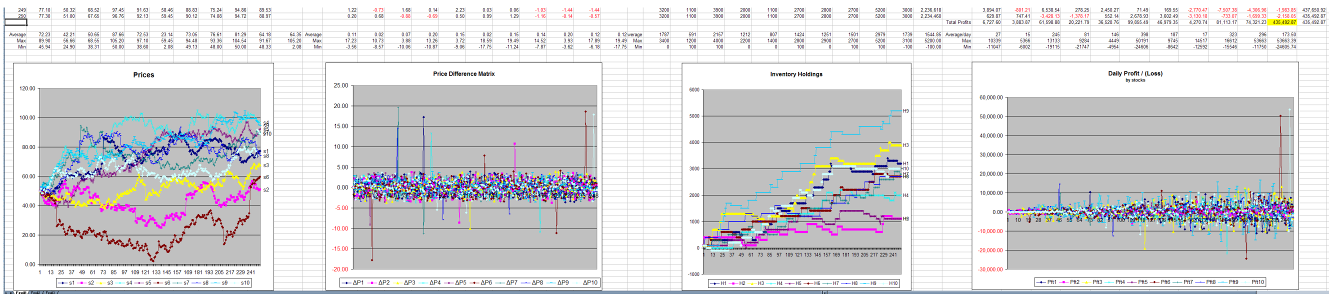
There are no predictions that can be made for any single stock in any one portfolio. Each price series has its own signature; its signal (drift) buried in its own noise. Each of the 2,500 price variations had about a 50% chance to rise or fall. There are some 15,000 calls to the random function in that file to determine what to do next: buy, hold, sell, short or liquidate.

The following Excel snapshot shows the first 27 trading days out of 250 for the **P**, Δ**P**, **H**, and **H**.*Δ**P** matrices. (Click the above link to expand the graph below.)



The **P** matrix is randomly generated. No predictions can be made better than a slightly biased flip of a coin. The Δ**P** matrix is simply the price difference from row to row of the **P** matrix. The **H** matrix is the trading strategy; it is what makes the difference. It is defined as **H** = **B** – **S**; the running inventory level of shares held. It holds the number of shares **B**ought minus the number of shares **S**old or **S**horted over time.

The last matrix: **H**.*Δ**P** is an element-wise multiplications giving the generated profits or losses from executing the trading strategy **H**. The next Excel snapshot shows the graphs of the above matrices with some statistics like averages, min and max values. (Click the above link to expand the graph below.)

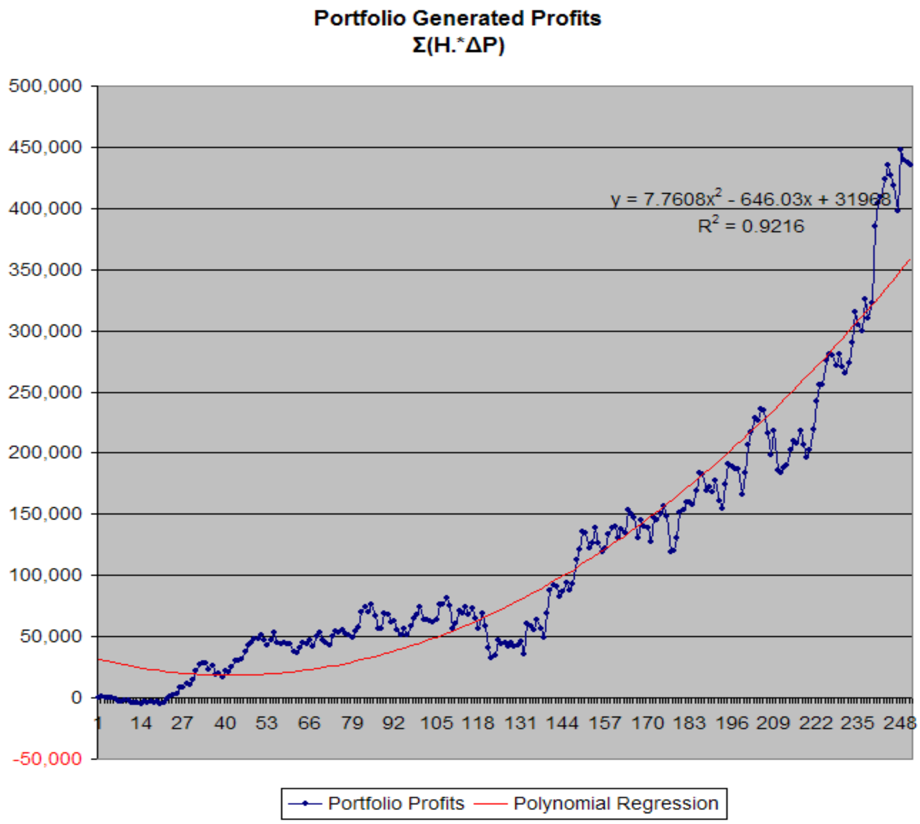


The graphs show the entire history of the **P**, Δ**P**, **H**, and **H**.*Δ**P** matrices. Of note: the outliers in the Δ**P** matrix and the increasing variance in the **H**.*Δ**P** matrix.

This would not be complete without a chart showing the performance results.

The blue line on that chart should have hovered around zero and have about a zero correlation coefficient. But it is not the case. It is saying that there is a definite trend. Its r-square value is 0.92 which can only say that the given quadratic equation (a power function) is a good representation of the generated profit line. This, in turn, means that the trading procedures used over the Δ**P** matrix are not only generating positive alpha but are also generating exponential alpha.

I know that I can not win on randomly generated stock prices. But even randomly trading the portfolio generated exponential alpha, an aberration in itself. Except if the market is **not** Gaussian. The Excel file demonstrated that point. And I think that is what that simple demo showed. I might not be able to predict where prices are going, but I can still design trading strategies that can randomly play the game and win.



The Setup

In the previous section, I presented the execution of a random trading strategy over randomly generated stock prices. The original intent was to answer someone on a LinkedIn forum on how to build a payoff matrix: Σ(**H.*ΔP**). So model 1 (very basic Excel file) was provided to show how to set up all 4 of the needed matrices: **P**, Δ**P**, **H**, and **H.*ΔP**. Each matrix dealing with an aspect of the payoff matrix used to simulate a portfolio of 10 stocks over 250 trading days (about a 1-year trading interval).

Closing daily price variations for the **P** matrix were generated using a simple random function: (Rand()-0.5)*4, which had prices vary within +/- $2.00. The Δ**P** matrix is simply the difference of closing prices from day to day (row to row) of the **P** matrix; an element-wise subtraction. The random price variations in the Δ**P** matrix would have, as expected, a mean tending to zero or close to it.

The **H** matrix: the trading strategy itself, contained the running stock inventory for all 10 stocks over the entire holding period. In that Excel file (model 1), the trading strategy was designed to be totally random and defined as **H** = **B** – **S**; the running inventory level of shares held. It holds the number of shares **B**ought minus the number of shares **S**old or **S**horted over time. The **B** and **S** matrices are of the sparse type having mostly zeroes for each of their respective elements and represent the trading decisions taken to increase or reduce the inventory as prices evolved from day to day.

As for the **H.*ΔP** matrix, it is the result of applying the strategy matrix **H** to the Δ**P** matrix; an element-wise multiplication. So the Δ**P**, as well as the **H.*ΔP** matrices, have little interest as one is a subtraction and the other a multiplication; they only serve in the bean counting process.

The two matrices **P** and **H** are the only ones of significance. The first one because it contains all the price history, the price at which trades can occur and the second one because it contains the trading history of what was done over the whole trading interval; it contains the history of all the trading decisions taken. And it is those trading decisions that in the end matter.

Each time F9 would be pressed in the Excel spreadsheet, a new randomly generated portfolio of 10 stocks would result. It would be the same as picking 10 stocks at random from an infinite stock universe. No two prices series would be the same, in a single portfolio or from one portfolio to the next. The worst kind of trading environment for a wannabe trader. You simply don't know what is coming next, therefore, on what should your trading decisions be based?

You could analyze the past data up to any point in time and nothing of it could be useful to predict the future. You could do data mining, pattern recognition, run any technical indicator, setup Fibonacci line, use stochastic or anything else, and nothing would help to determine what to do next. No software package, what ever its charting abilities, could help predict the outcome of a random price series.

Naturally, with such a trading environment, using model 1, you could not win except by luck alone. You were after all using randomly generated trading decisions over randomly generated prices. It would be akin to flipping a coin to bet on someone playing heads or tails. Sometimes you would win and sometimes you would lose. The expected outcome of the game would tend to zero. And if a payoff matrix tends to zero, then you have a zero-sum game where your expected value is the same as everyone else: zero.

There is no free lunch to be had; there is not even one available. The only optimum portfolio is to achieve the same average as anyone else, and that is zero profits. And thereby no alpha generation; no over-performance.

Changing to Model 2

You now switch to model 2 where prices are allowed to fluctuate even more than in model 1. You add more random components including “fat tails”; low probability events of higher magnitude. Instantly, prices become more erratic and even more unpredictable than in model 1; you have introduced “outliers” rare events in your already randomly generated price structure. You are introducing improbable directional “gaps” in the price data series. Again, nothing will help “forecast” what the next day may be.

In model 2, the price series for any of the stocks has become: initial price to which is added a low drift and the sum of random-like price variations of varying magnitude and with varying probability of occurrence. This is the same as having a random process with drift, diffusion and jump components. Model 2 also resulted in a more realistic representation of stock prices series.

By introducing added randomness to the generated prices series, you have not simplified the trading process but have definitely added another level of complexity to the trading decision process itself. How could you now trade knowing that your trading risk has greatly increased? An unpredictable major gap down could result in an immediate haircut and therefore will require a change in the trading methodology, especially in the bet sizing department.

But even with the added risk, a randomly generated trading strategy would remain with a zero expected outcome. All you would see would be more unexpected price variations, more volatility, that you still could not predict and with no way to knowingly establish a position before the price gap up in order to profit or save you from the loss of a random gap down in price. All you could do would be to speculate, guess or take your bet based on whatever concept might suit your fancy. You would be left trying to outguess randomness, by whatever method of your choice. And therefore, your methodology, whatever it may be, would be as good as anyone else's. We would all be playing in a quasi-random trading room. Anyone could have a series of lucky moves up or disastrous gaps down.

Changing Trading Habits

The whole trading process was being modified by adding more randomness to the design of model 2. It was still randomly generated but acquired some conditionals: some do it only ifs. And it is those trading rules that make the difference. In model 2, you pressed F9 to generate a totally new portfolio not only in prices but also in the conditionally generated strategy and you would win almost all the time (over some 98%).

Model 2 was very easy to construct, it is a downsized version of a similar setup used in my first 2007 publication: **Alpha Power**. In that publication, the payoff matrix was for 50 stocks by 1,000 trading weeks. That paper was followed by the **Jensen Modified Sharpe** in 2008 where the payoff matrix had for size: 100 stocks by 2,000 weeks. Each time you would press F9, the payoff matrix would show exponential growth; generating exponential alpha.

The whole financial investment literature says that you can't win over randomly generated prices, and most certainly not by using randomly generated trades (conditional or not). But here, a simple Excel file says: yes you can and with relative ease. A low long-term upward bias in price coupled with a long-term view of the investment process would appear to be sufficient.

One could switch to model 3 which has a better trading philosophy (meaning also having higher profits) and win 100% of the time. Not that one would win on all the stocks in the portfolio or on every trade but still win at the overall portfolio level. Because of the randomly generated price series, about half of the price variations would still see red on any given day, and the portfolio would see the same relative losses and gains. Look at the model 2 charts where red appears to be randomly distributed in the Δ**P** and **H.***Δ**P** matrices.

The Implications

What are the implications, what can we learn from model 2? The main output of the payoff matrix Σ(**H.***Δ**P**) is an exponential function and this leads to exponential alpha generation. It should have, on average, tended to zero just like in model 1, but instead, even if I ran it a thousand times, the model's payoff matrix remained positive and exponential.

There is a need for a reasonable explanation for the behavior of model 2. There is a need to find a mathematical model that can explain the phenomenon. And if the explanation is reasonable, backed by simulations on these randomly generated prices and conditional random trading decisions, then the foundation of a “new” trading methodology might be settling in.

Changing the Game II

In the previous section, it was presented that even trading randomly over randomly generated stock prices could not only generate a positive outcome to the portfolio payoff matrix but that this outcome could generate exponential growth.

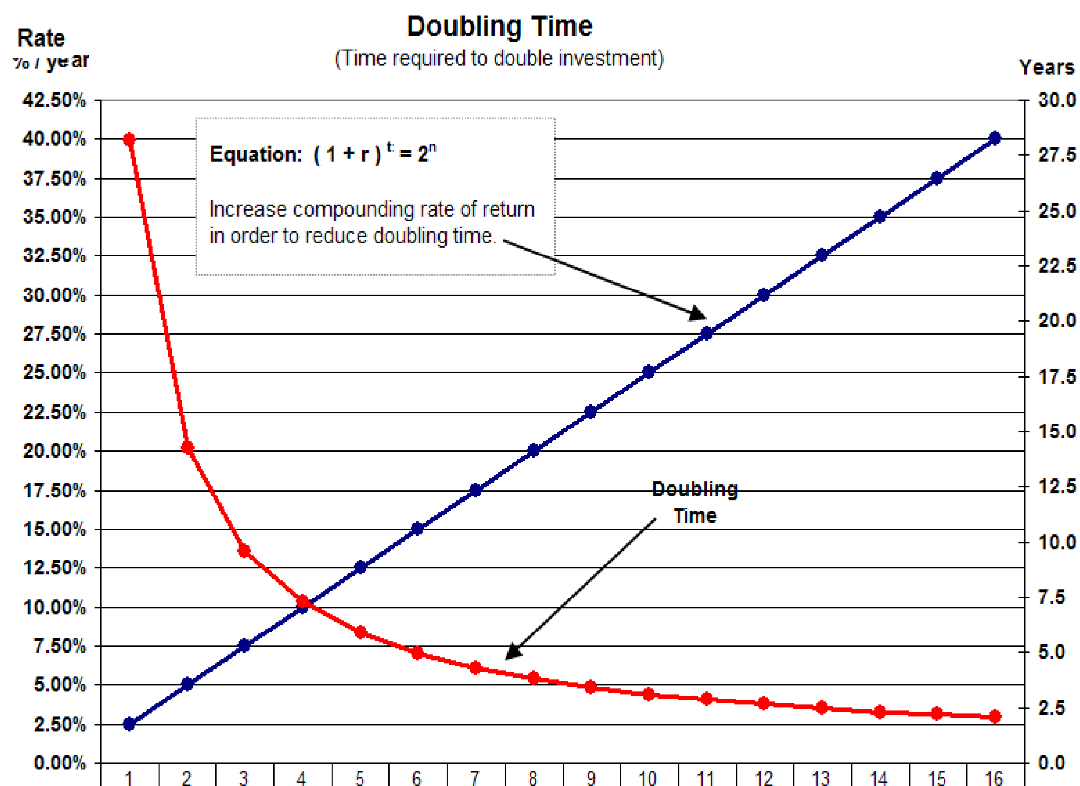
To some, it is unthinkable that a trading strategy governed by randomly generated trades over randomly generated stock prices (including unpredictable gaps) could have profits on an exponential growth rate or even a positive growth rate for that matter. As was said before, the expected value of using heads or tails to determine some other heads or tails' bet is zero. Except if one or both coins are slightly biased.

Yet, all these randomly generated trading strategies trading over randomly generated prices were producing profits at an exponential rate. This is a major statement: it would imply that as you progressed in time, the rate of return would be increasing in time at an exponential rate. It does not go fast, but progressively, it starts at zero and then grows and will continue to grow. As a side effect, it will gradually reduce doubling time; one of my highest objectives, if not highest priority.

Doubling Time

The stock market game is a compounding rate of return game. The main objective is to obtain as high a rate of return over the longest time interval within portfolio constraints; the first being not to go bankrupt. In its simplest form, the output of the game could be expressed as FV / PV = $(1 + r)^t$, where FV is the future value of the portfolio while PV is its present value.

The following chart illustrates the relationship between r and t when analyzed to determine how much time is required to double the initial investment.



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	
Rate of return	2.5%	5.0%	7.5%	10.0%	12.5%	15.0%	17.5%	20.0%	22.5%	25.0%	27.5%	30.0%	32.5%	35.0%	37.5%	40.0%	
Doubling Time	28.16	14.25	9.60	7.29	5.90	4.97	4.30	3.81	3.42	3.11	2.86	2.65	2.47	2.31	2.18	2.06	Years

For instance, a long-term compounding rate of 10% would require about 7.29 years to double one's portfolio. It would require 7.29 years to double again, and yet another to double again. Some 21.87 years to have one's portfolio grow by 8 times.

The first 10 numbers of the doubling series have the following progression: 1, 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024. As one progresses along the series, it might take the same time to reach the next level, but this new level represents the same as was done for the entire preceding levels of the series. To reach the 10^{th} level (1024), would require some 72.9 years at the 10% rate of return, but only 26.5 years at the 30% rate.

The above chart does illustrate that much effort should be put on reducing the doubling time as much as possible, and this within all the other limiting factors one might encounter while implementing a long-term trading strategy. For the game is not a one period game, it is a multi-period, multi-asset long-term game. And the real goal is not the first doubling, but the latest one in the series. You will double your initial stake in the first doubling time, but from the 9^{th} to the 10^{th} level, you will increase it by 512 times; as much as all the previous 9 levels.

IMHO, the real “holy grail” of investing is there, at the other end of the rainbow. And as funny as it may seem, it is built-in the game. All one can do is try to reduce as much as possible the doubling time and persevere long enough to reach the 10^{th} doubling. But one can reach the 10^{th} doubling if and only if he/she was able to reach the 9^{th}, the 8^{th} and so on. It is a long-term process and someone should be armed with patience, know-how and determination.

P and H Matrices

Since from the previous section, we were left with only the **P** and **H** matrices having any significance, all the efforts should be concentrated on better understanding their respective behavior and interactions.

The price matrix **P** is simply the set of daily closing prices of the selected stocks. It could be of any size; one could select the 100 stocks of the S&P 100 for example, and easily build the **P** and Δ**P** matrices for 250 trading days (1 year) or 5,000 trading days (20 years).

One could also use randomly generated prices that could mimic as much as possible the Paretian nature of price movements, meaning data series having an initial price to which is added a low long-term drift, some random price variations plus jumps (price gaps, outliers). This makes prices totally unpredictable. Especially if the random price variations buried any signal there might be in excessively noisy data; to the point of containing “black swans” generated by the up and down price gaps.

You could eventually predict the drift, but only after a long time and for the long term. Even if you had knowledge of the drift, it wouldn't help in the day to day operations as it would be literally drowned in all the noise and with a figure too low to really profit from it short-term compared to the risk taken.

There is the conundrum: short-term, prices behave as if quasi-random; one is almost making a bet, gambling that prices might go up or down. Even if the long-term drift is up, tomorrow's price may see a down day; and with low probability a major gap down. Because the drift is so small, short-term, it goes undetected and probably undetectable. And when looking long-term, the drift will tend towards the average market return. As a consequence, no long-term alpha generation, or if any, an alpha tending to zero from either side of the market average.

Again The Implications

It is easy to set up the **P** and Δ**P** matrices for backtesting the S&P 100. All the numbers are out there. You can analyze past data series from every angle using any simulation software package available. However, all past data will remain just that: past data.

Backtesting on the 100 stocks of S&P 100 index would be like always back testing on the same thing, the same data series over and over. It is very easy to have one's backtesting program adapt to the set of data series at hand. The Δ**P** in the payoff matrix will not change, it will remain the same from test to test. Designing a better trading strategy $\mathbf{H}^+$ becomes a matter of finding a better inventory averaging process for the S&P 100 Δ**P** matrix: $\Sigma(\mathbf{H}^+ .* \Delta\mathbf{P})$. It is the same as “adapting” the trading strategy $\mathbf{H}^+$ to what was.

Backtesting on simulated market data made to mimic market prices, as in the randomly generated prices discussed above is something else. There, random price variations with drift and jump components become unpredictable with very little serial correlation. Using randomly generated data can lead, I think, to a better understanding of certain trading procedures, especially if having a long-term view of the market. The main advantage of using simulated data is that the $\mathbf{\Delta P}_?$ matrix will always be different for each stock and for each portfolio from test to test. There will be no short-term predictive price relationships to be found. Having a totally different $\mathbf{\Delta P}_?$ matrix each time a test is undertaken will force to consider the trading strategy **H** in a different light.

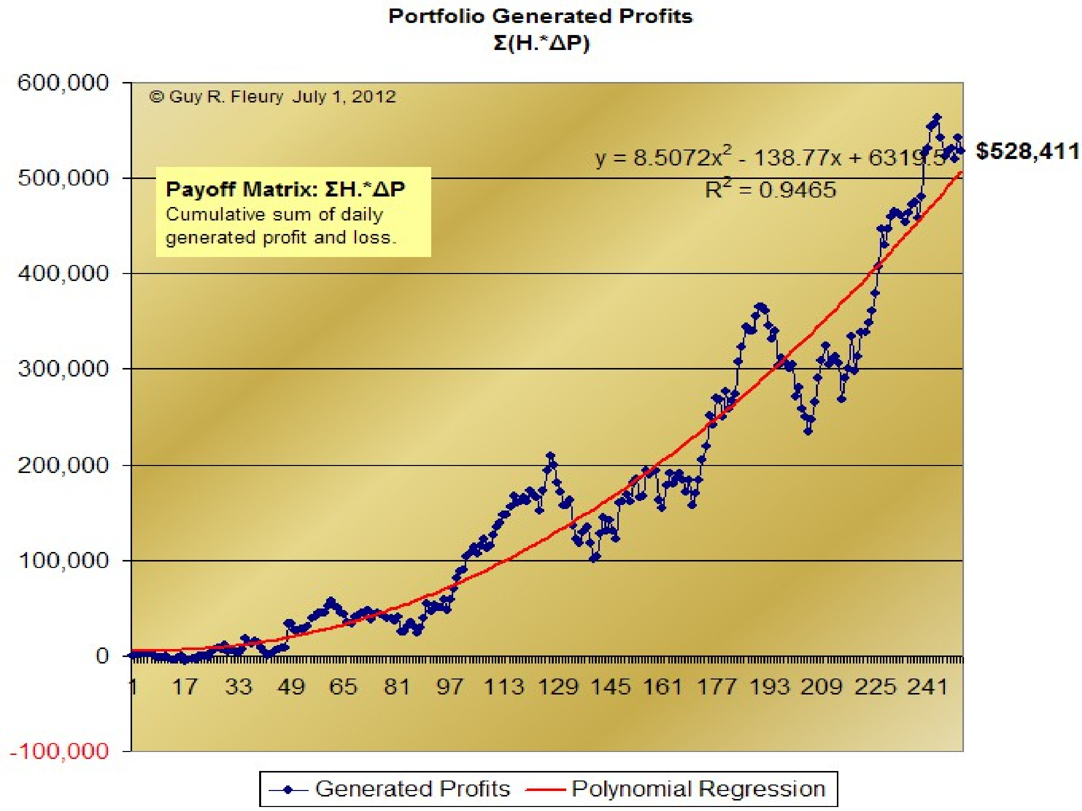
Much research has been done on past price data. Do stock prices, for instance, maintain some memory of their past? Which indirectly says stock prices can trend or display auto-correlation. One thing is sure, the longer the time horizon, the higher the probability that stock prices have, on average, an upward bias. One will find over the past 200 years, that over 95% of 20 years rolling trading windows are positive for the US market. This is the same as saying that over a 20 years investment interval, one has about a 95% chance, on average, of ending with a profit. But even knowing this, we still have only one future ahead of us.

If you trade using the output of random functions on what you estimate as randomly generated stock prices that also mimic price jumps (gaps) how could you generate positive returns, wouldn't that be like a contradiction in terms? Just the general notion that the long-term trend (low drift) could be positive would appear as a sufficient criterion for predicting long-term prices. And if the price series are generated using random functions, how can anyone predict, short-term, what will come next?

To demonstrate my point, I designed such an alpha generation strategy. I opted to provide an Excel file showing the principles at work. This is a much smaller design than in 2007-08, where 2 other Excel files, one a 50 x 1,000 and the other a 100 stocks x 2,000 trading weeks were developed using more elaborate trading procedures than those presented in this file (a 10 stocks x 250 trading days).

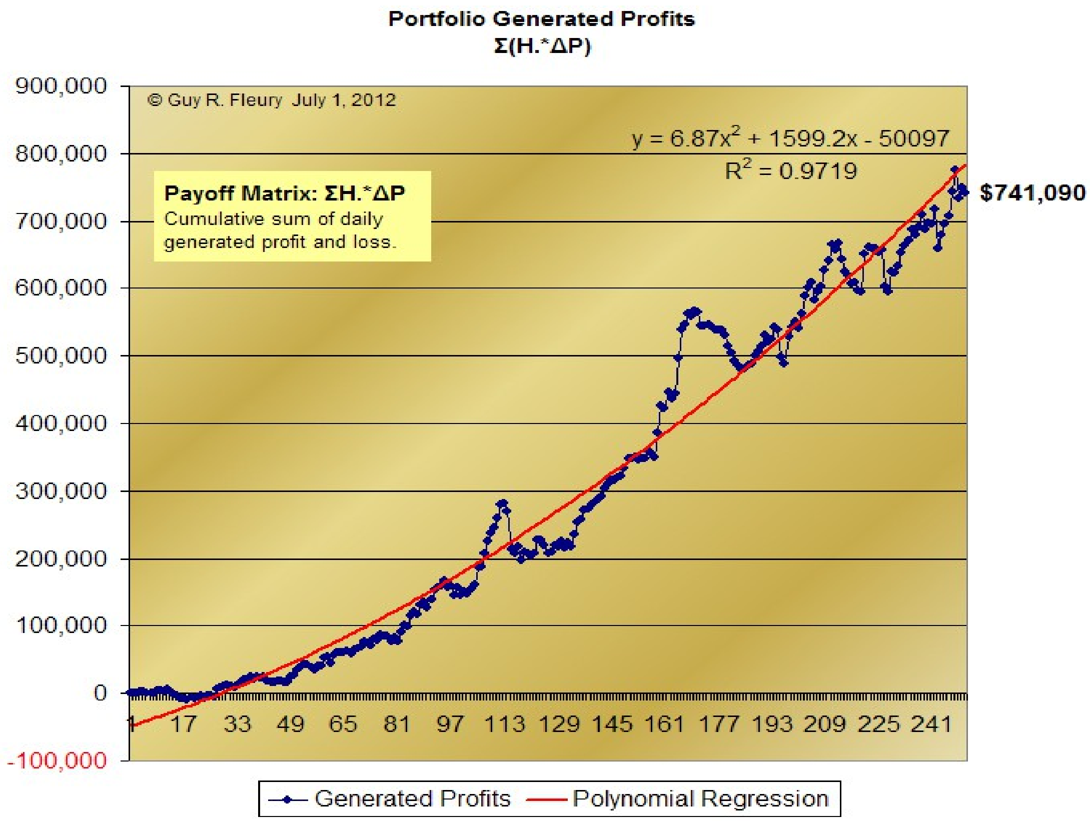
Alpha Generation Excel File

The conclusion of the Excel file can be showcased in a single graph:



The outcome of a trading strategy **H** applied to a price difference matrix **ΔP**? resulted in the above payoff. The two main aspects for this payoff curve are: first, generated profits increase in time; and second, to a high degree of correlation, this increase in time is quadratic, it increases at an exponential rate.

In Excel, to execute a new test, it is sufficient to press F9 and every formula is re-calculated. Having prices as well as all the 2,500 trading decisions randomly generated, no two portfolios could ever be the same from one test to the next. For instance, pressing F9 again produced the following:



Both graphics show exponential growth and are the result of applying a random trading strategy to a randomly generated price difference matrix. The trading strategy, being randomly generated will buy about anywhere during the trading interval without regard to what the price may be at the time. And on the same principles will also sell or short shares based on randomly generated trading decisions.

Generating Random Stock Prices

Real stock price variations are complex, to the point of being hard to predict. And the harder they are to predict, the more one should consider that randomness plays a major part in the price's evolution. If randomness wasn't part of the equation, predicting stock prices would a lot easier.

The following equations were used to model and generate random-like stock prices:

$$P(t) = P_o + \Sigma\Delta P$$

$$\Delta P = \text{Drift} + \text{Random variations} + \text{Gaps}$$

The first equation says that price is a function based on an initial price (P_o) at time t=0, to which is added the sum of all price variations thereafter.

The second equation says that the price variations are composed of 3 elements: a long-term drift to represent the secular market trend (20 years or more) to which is added a purely stochastic process and some outliers (gaps) as unpredictable price jumps.

The Excel equations to generate these numbers are easy to construct. The drift is simply a number representing the slope of the long-term trend expressed in pennies per day (see cell D3). The random price variation is produced using the Excel rand() function which returns a number between 0 and 1. By subtracting 0.5 from the rand() function makes it vary between -0.50 to +0.50 and multiplying this output by another number will determine the amplitude of the random fluctuation: like (Rand()-0.5)*3 for price variations ranging between +/- $1,50.

Generating price gaps is as easy as generating any other price variations. They are simply low probability outcomes of higher amplitude. Again, using the Rand() function:

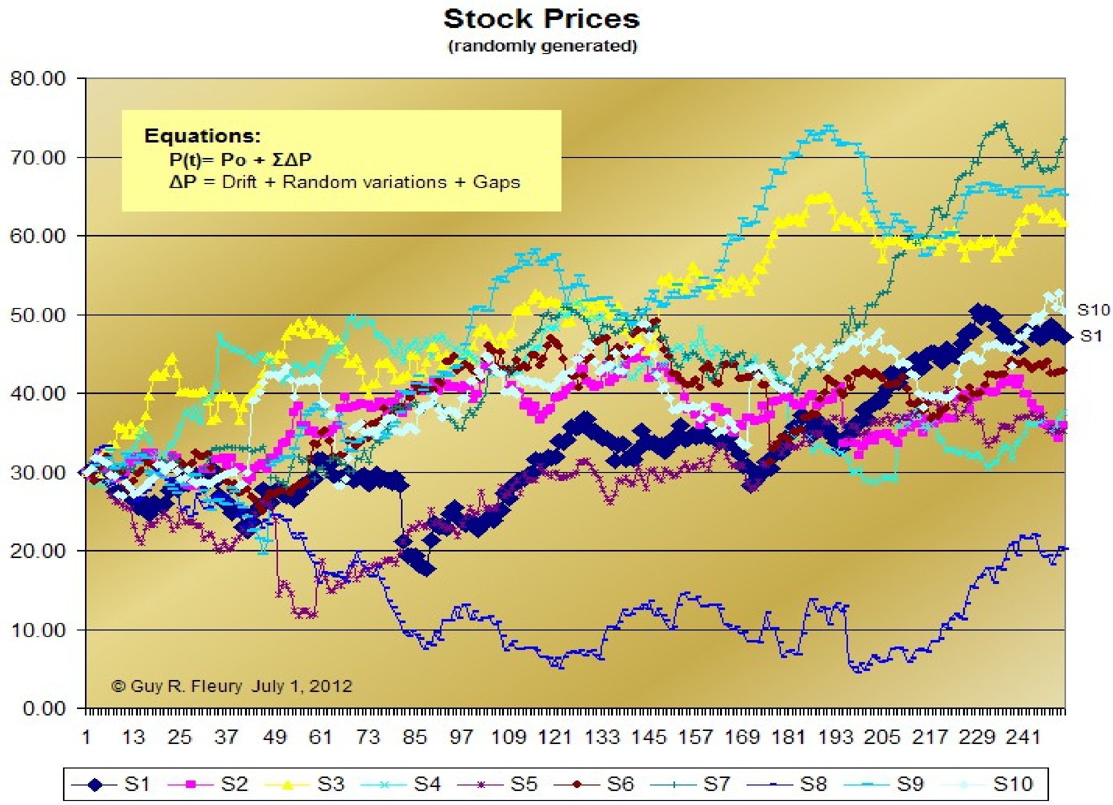
+ If(Rand()>0.99, GapMultiplier*(Rand()-0.5)*10, 0)

translates to: you have 1 chance in 100 to generate a price gap ranging between +/- 10.00 provided the GapMultiplier is equal to 2.

All this is relatively simple stuff. However, the output as shown in the price matrix is interesting. The price matrix becomes totally unpredictable. You can not know what will come next, either a small price variation up or down; or once in a while, a “black swan” could also come your way, up or down.

The 10 price series generated are like picking 10 stocks at random from an infinite stock universe. There is nothing you can learn from one data series to help you determine another. And there is no knowledge gained in a single test that can be transported to the next.

A view of the price matrix as a whole looks like the above chart. Pressing F9 in Excel will generate all new price series for each of the stocks in the portfolio. From any point in time of the generated price series will there be any type of statistical or technical analysis to help determine what is coming next.

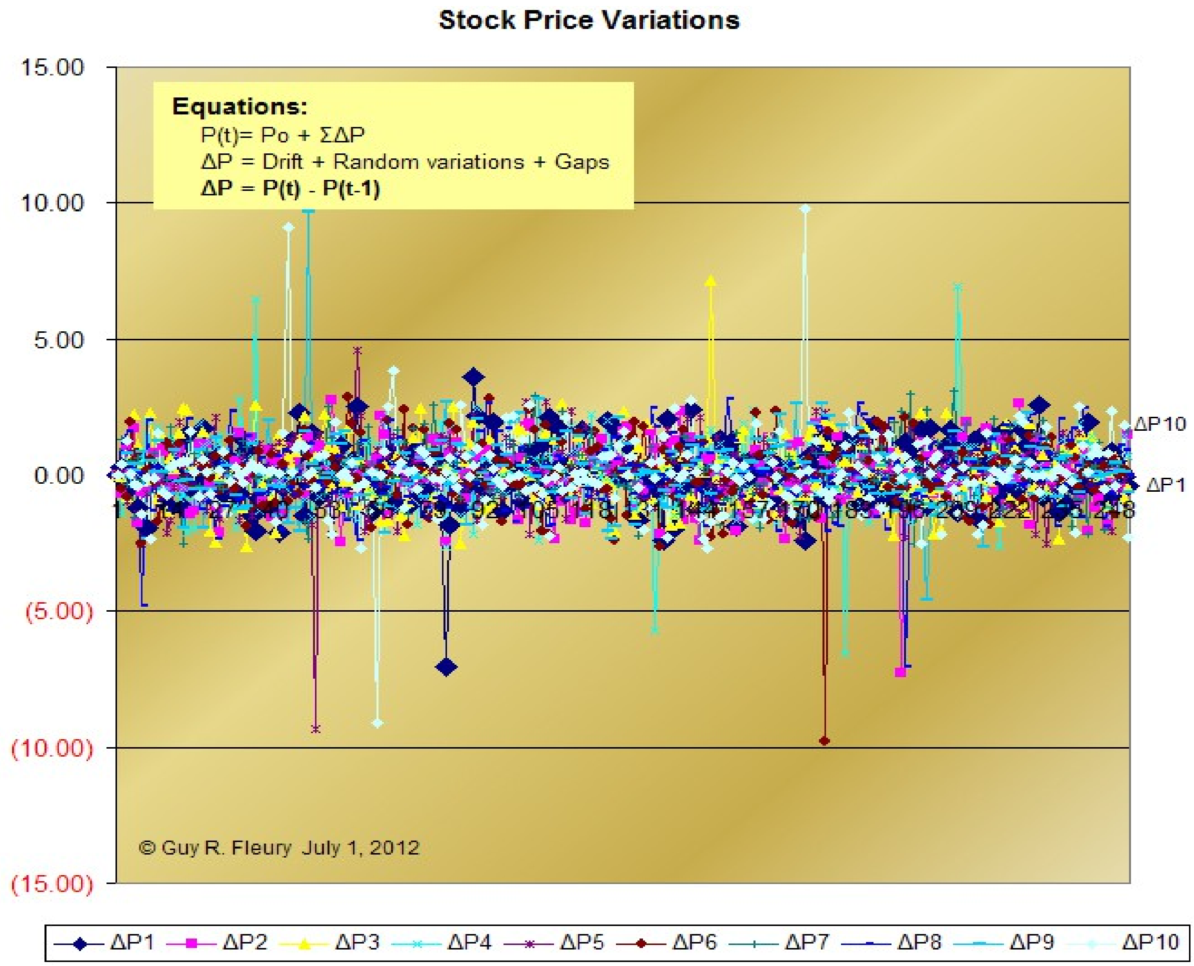


The Price Difference Matrix

The **ΔP?** matrix is the result of a simple subtraction and should present little interest. It is nonetheless required for the calculation of the final payoff matrix. Its equation is: Δ**P** = P(t) – P(t-1). The Δ**P** matrix is the difference in price from row to row (day to day) of the **P** matrix; another way of expressing the variation in price from close to close.

Of note in the price difference matrix are the outliers (gaps) that occur at random times with random amplitude. They tend to mimic what we sometimes see in real market data; large unpredictable price moves (gaps).

The price difference matrix Δ**P** shows all 2,500 price variations:



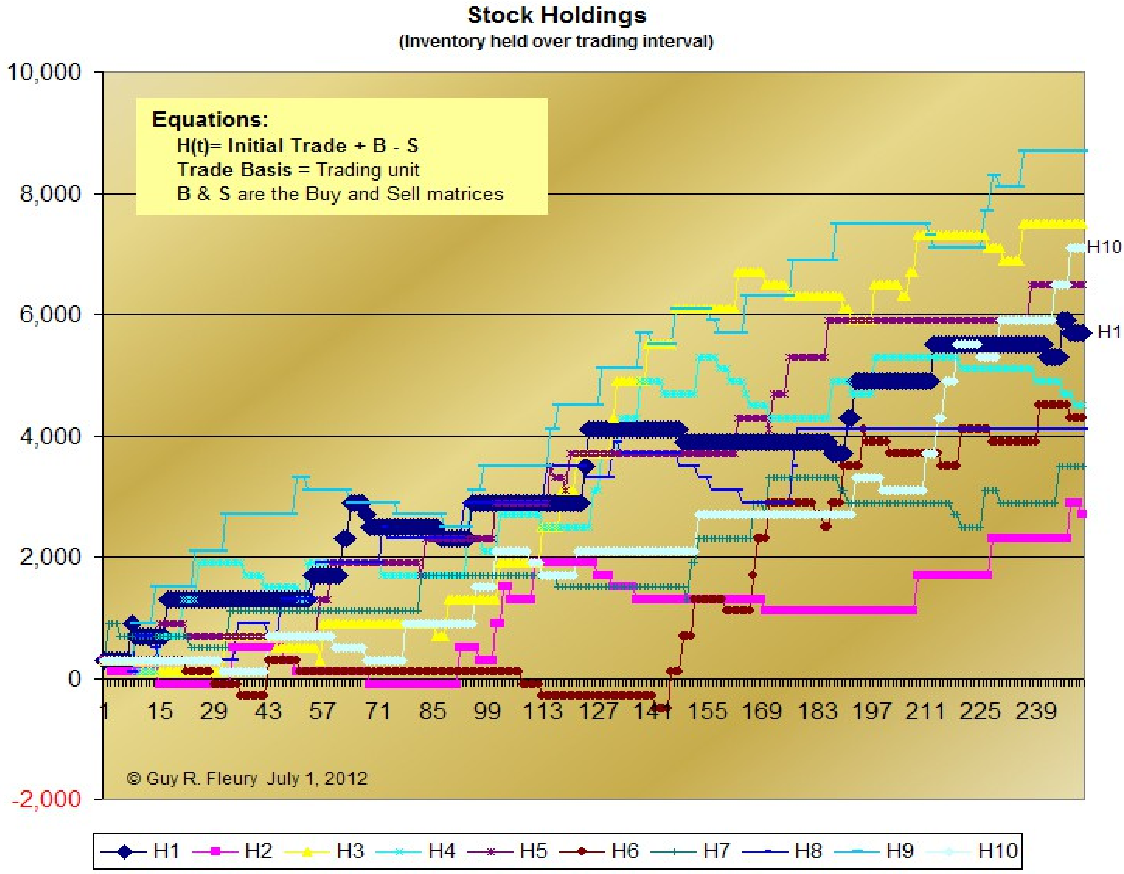
The Trading Strategy

Here is the crux of the whole spreadsheet: the trading strategy. It is randomly generated over the entire trading interval, in this case, with 250 trading days, will require 2,500 trading decisions to be made.

The basis for the equations used to generate the trades can be found in my 2008 paper: **Jensen Modified Sharpe** from page 28 to 35.

From their respective initial positions, shares are bought and sold throughout the trading interval. A trading unit (number of shares to be traded at on time) serve as trade basis and is used for both long and short positions.

A representative stock holding matrix looks like the following:



As can be seen, there is a tendency for the inventory to rise since the objective is to accumulate shares over time. By disabling the shorts, the inventory levels would only rise and look more like Lévy processes with random jumps.

For longs, the governing formula for this random trading was:

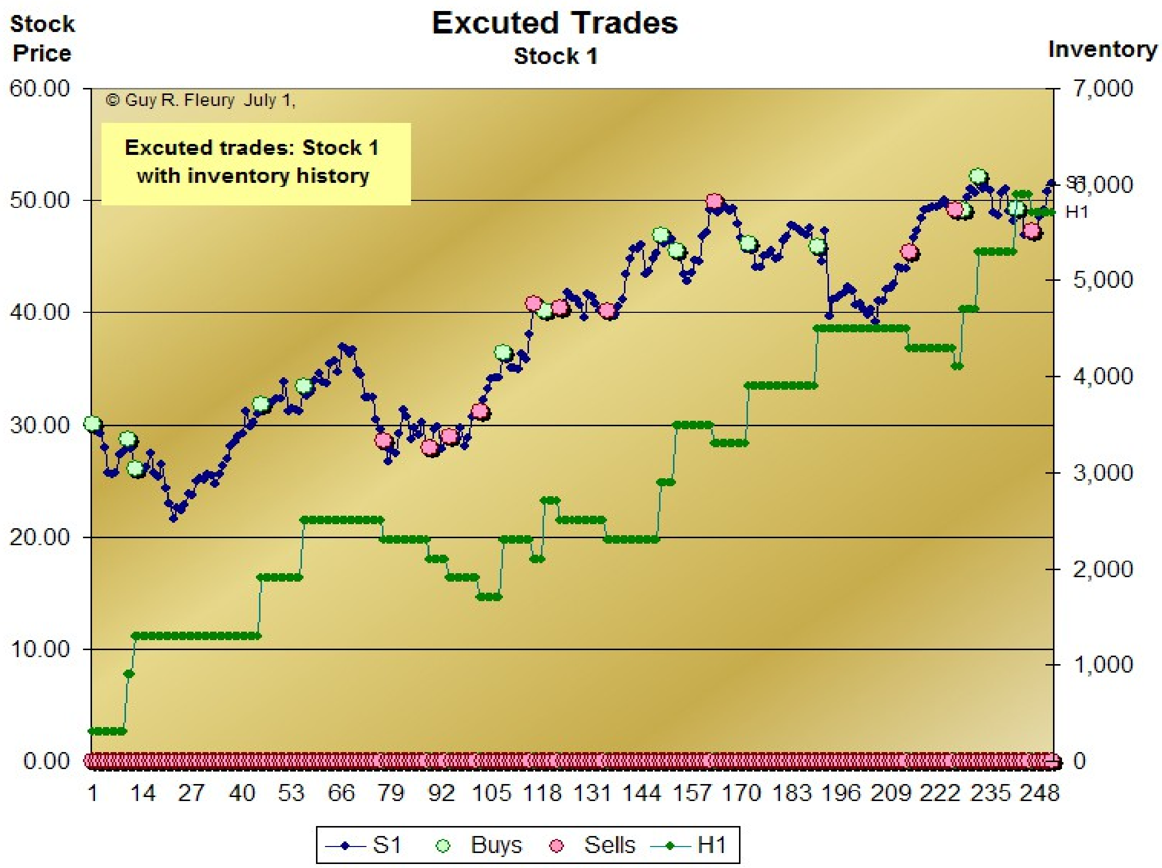
+ LongMultiplierEnabled * IF(Rand()>0.95, TradeBasis, 0)

which will generate 1 unit trade, on average, over a 20 day period. The shorts operate under a similar principle: + ShortMultiplierEnabled * IF(Rand()<0.05, -TradeBasis, 0).

It is by creating an imbalance between the longs and shorts trading commands that following the initial purchase, each stock will tend to accumulate shares over the trading interval. And this inventory accumulation process will translate into profits as was seen in the generated portfolio profits chart presented above.

By accepting an accumulative inventory process, there is a Buy & Hold dimension that is being developed over the trading interval. Adding positions is like providing a reinforcement feedback over the trading pattern.

A typical price series with its trading history is provided in the following graph:



To show the principles at work I opted to have a relatively high trading rate. But this could also be controlled to any desired level. All the price variations are random including gaps (fat tails). And all the trading decisions are the outcome of random functions.

The Payoff Matrix

The payoff matrix is a simple multiplication. It is the result of applying the holding matrix **H** to the price difference matrix Δ**P**. The payoff matrix shows the daily profits and losses generated as time advance. Since the **P** matrix had the ability to generate outliers, these can be found in the large drawdowns that appear here and there over some of the price series. There are also up-gaps as can be seen in the graph. None of those outliers (gaps up or down) could be predicted.

The Portfolio Generated Profits

The last chart in the spreadsheet and probably the most import is the Portfolio Generated Profits graph. It is simply the cumulative sum of the Payoff Matrix: $\Sigma(\mathbf{H.*}\Delta\mathbf{P})$.

In the Excel file, pressing F9 generates a new scenario. In a final analysis, this graph is the only one of interest; it says how much was won or lost for that particular randomly generated trading strategy as applied to the randomly generated price series.

No two payoff matrix will be the same. But that is not important, it is expected. As a matter of fact, I don't think that anyone in a billion lifetimes will ever generate the same **H** and **ΔP** matrices as someone else. The future is unknown and this payoff matrix is surely a representation of this.

What is important however is that the best regression line to be applied to the data is a quadratic equation. Often with an r-square over 0.90 showing that there is a pretty good fit for the underlying data. And having a quadratic equation as the best explanation for the data implies that the data itself is an exponential function; in this case, a growth exponential function. It should be noted that a third-degree function has a higher r-square figure, but I did not want to crowd or complicate the issue.

Each time the F9 key is pressed, a totally new trading strategy is applied to the newly generated price series. Yet, the output of the payoff matrix $\Sigma(\mathbf{H.*}\Delta\mathbf{P})$ remains positive. All the academic literature over the past 60 years says that randomly trading over randomly generated prices leads to zero alpha. And here you have this simple Excel file, not only generating alpha but exponential alpha.

Intermediate Conclusions

All the above not only shows that one can produce alpha, even on randomly generated data, but that even using randomly generated trading strategies; this alpha will be of the exponential type. It implies that the rate of return will increase in time. Starting at zero, this rate of return will increase at an exponential rate. This also means that the doubling time will decrease over time, gaining the ability to gradually reduce the time it takes to achieve the long-term objectives.

It is not about achieving 100% per year from year one, it is about gradually increasing the rate of return to even exceed 100% as time progresses. It is in the nature of compounding returns and in the formulations presented in my original papers and in all my webpage notes for that matter. All my simulations also make the same point: accumulate and trade over the accumulation process. And by adopting such trading procedures your own trading strategies will also get a boost.

I don't advocate trading randomly, there are much better trading methods that can do more than what this simple model 2 Excel file tries to demonstrate. However, it does show that provided there is a long-term upward bias on average in stock prices, it is sufficient to simply accumulate shares over the whole trading interval to increase performance results.

Changing the Game III

After the conclusion of the previous section, it is time to put out the Excel file that generated all of the above. I think it will enable anyone to “play” with model 2. This file is a working model designed to showcase some basic trading principles and methodology. It is not an end-all, but it does show that accumulating shares and trading over this accumulative process can generate profits even if the entries and exits are taken at random.

May I offer the **Excel file** that generated all the charts in the previous section (this file no longer made available, too much proprietary stuff).

The file generates random stock price variations to which is added a drift and price gaps (fat tails, outliers). The trading strategy uses random functions to generate unpredictable trades, without knowledge of prices, past or present, during the whole trading interval. It is all explained in previous sections: **Changing the Game I** & **II**.

No one wants to flip a coin to trade every day! It is not the nature of the game. Nonetheless, you need some Δ**P** to make a profit and that takes time or predictive skills.

One should consider that applying some of the same trading principles where prices can have some predictability, even at a low level, and where you can predetermine the kind of trading strategy that you would want to implement should easily produce better results. Many of my simulations show just that.

Using random-like trading might not be the most efficient method of trading, even if it does produce profits. However, I think, it can provide some insight on improving one's own trading strategies.

The Excel file highlights the concept that accumulating shares over many stocks over a long-term investment period might be sufficient to generate some alpha. And this accumulation process, over the trading interval can be created using a simple trade imbalance. It's like following the trend, accumulating shares as prices go up, sell some of the shares, and go back to accumulate, on average, some more shares (see the Stock Holdings graph).

The trading game is more like an investment game than a gambling game. One needs a long-term vision: a what will happen as time goes by? I expect that given more time the strategy provided would simply continue to trade over its accumulative process. Using a 1-year time interval only shows part of the long-term objectives designed in the trading procedures.

Notice the graph “Executed Trades” in the previous section, where some shares are sold at the bottom while others are bought at the top. But overall, still generated some profits due to the accumulating inventory.

Some Operational Notes:

The spreadsheet is not restricted in any way. It is intended to serve as a basic working model; a framework to test your own ideas using your own trading methods. IMHO, it should raise a lot of questions on the trading methods we use. There is no fundamental data here, no technical analysis, no statistical analysis, and no astrology measure. Just plain random price data and plain random trading strategies.

Red cells and cells with red triangles in their upper right corners contain additional information pertaining to the selected cell or its surrounding. This Excel file should be considered only a beginning, a crude example of what can be done. I think, that it should lead to considering other ways of designing trading strategies $\mathbf{H}^{+}$ that better suit one's own objectives, requirements and portfolio constraints.

Recommendations: start by making a copy of the file in order to keep all the default settings intact. This way you will be able to explore, make changes and keep the ability to return to a known state. As you advance, again make copies of your improvements to later compare how your own methods evolved.

This is not a game with a single answer. But one thing I am sure is that as you modify this Excel file, you will be making it your own and most importantly understand what is going on behind all the equations to make it do what you want.

The spreadsheet is about 65 columns by some 340 rows. May I also suggest some exploration and a lot of pressing F9 to get familiar with all that is happening in the file. There are side effects to the equations used that need to be understood before making modifications to the formulas.

Start by making changes to the yellow cells which control the trading environment. For example, increasing the initial trade quantity will have for effect not only to increase initial required capital but also enable reaching auto-financing earlier, and from that point on, have the market pay for the accumulative process in the sense that the accumulating profits are paying for the new purchases. A simple Buy & Hold scenario only needs to set the trade basis and the sell enable to zero.

Continuously pressing F9 will generate test after test, and if located near a cell or a chart that displays the portfolio's generated profits, will show that a negative number does not happen often. And if you modify some of the formulas, your goal should be the same: no or a low number of negative results.

The Portfolio Generated Profits chart (on the far right) shows the net result obtained by trading strategy **H** applied to the Δ**P** matrix. It is the total profit or loss left after liquidating all the stocks in the portfolio. Your objective is to keep it positive and exponential as much as possible. Below the Portfolio Generated Profits chart, you will find the Executed Trades graph. It shows, for stock #1, how trades were distributed as well as the inventory level as it changed in time.

The Trading Environment

In its default setting, the trading strategy **H** creates a trade imbalance: its buys 3 units while only sells 2 at a time. The independent **B**uy and **S**ell functions are randomly generated with a probability of about once a month. Therefore, over time, shares should be accumulating at a rate of about 1 unit per month.

Such a strategy resembles a kind of dollar cost averaging method where trades are not set on dates, but at random times; and it is more like fixed quantity averaging. The residue of the trade imbalance, the accumulating inventory, serves the same purpose as averaging into a position.

This makes this particular strategy a long-term proposition. It is not a day trading method and is based on a long-term view of the market. Technically pen, paper and a quarter might be sufficient to implement such a trading method.

The price matrix is structured to have random price variations of varying amplitude, with unpredictable price jumps and a long-term drift. All prices were normalized to the same starting point without loss of generality. This way they could all be treated the same. And since prices are randomly generated, it is like picking 10 stocks at random from an infinite stock universe. Each time you press F9, a new independent future is generated.

The trading strategy has a long-term view of playing the game, therefore, it should sound reasonable to also select stocks with a long-term view of the market. Prices being quasi-random, they have been made to resemble a Paretian distribution with fat tails. They can exhibit trends of various durations, cyclic moves and patterns of all sorts. The long-term view in the price structure is expressed with the low drift value. Random price gaps (outliers), which have an average frequency of about 20 weeks (100 trading days), and serve as rare events; a kind of warning sign to play it safe.

Application In The Real World

In real life, one would select a stock to be included in his/her long-term portfolio based on their long-term view of that particular stock. If you want to do a kind of constant volume averaging (a long-term strategy) might as well select stocks that you think might go up long-term. And if after some time, you find that your selection will not go where you thought it should, then liquidate the position, accept the loss and move on. Make another long-term selection. And since, by principle, the method over-diversify by having 50 to 100 stocks in its portfolio; any one bad selection will have little impact on the overall portfolio.

The 2008 **Jensen Modified Sharpe** paper provides some of the mathematical background for the trading strategy presented here. Especially pages 28 to 35 which dealt with the equations pertaining to initial trade and trade basis.

It should be noted that in the 2008 paper, prices might have been randomly generated, but the set of trading rules and procedures were not.

Over the past 4 years, all these methods have evolved and improved as I gained a better understanding of the forces at work, and designed better mathematical structures where I keep a long-term view of market prices. I still can not predict what the future will be but I am ready to take the bet that it will lead to better things, not worst.

There are many ways to improve the trading strategy and performance levels in the Excel file. One could add matrices to help achieve better trading decisions or set other matrices to control the trading behavior, set information matrices to analyze prices or add indicator matrices of all sort. One could also expand the matrices to say 100 stocks by 2,500 or 5,000 trading days. This last point would help better understand the long-term effects of the trading procedures used.

The Excel file model 2 demonstrates that using randomly generated prices, one could trade profitably even if trades were randomly generated. It was sufficient to have a long-term upward bias in prices and that there was a share accumulation process over time. Both easy to implement.

But still not the best way to trade.

I found that applying some random entries could help improve long-term performance level; but still cannot justify a random exit (there is an opportunity cost involved here). I explained this somewhere like this:

We seem to view the trading world as traders and investors with a great divide between both valid approaches. When in fact, they just operate at opposite ends of the time spectrum.

What I see is that the long-term investor has an expected value of doing about the same as his/her siblings. Whatever the next 20 years may bring, their most probable outcome will be to do about the same as everyone else who adopted the same long-term premises. You can view the end results as a compounded rate of return:

$$[\text{Capital}_t / \text{Capital}_{t=0}]^{(1/t)} - 1 = r \rightarrow \text{long-term average market return}$$

That you be a short or long-term player, you will be subject to the same equation. The ending capital compared to the initial capital can be expressed as a compounding rate of return over the investment period for any type of market player.

The real question is: who will have the highest rate of return *r* over the same time interval? Which trading or investment method will provide the higher r over the long haul?

Trading or investing is not just a one year endeavor, it should be a commitment for 10, 15, 20 years or more. And there lies the real problem to be solved. In the beginning years, all methods might be different and operate at different rates of return but when considered over the long term their respective efficiency might diverge and by a considerable amount. You are playing a compounding return game and a few percentage points over the long-term horizon can make quite a difference in the dollar amount earned.

Academic papers over the last half-century have more than elaborated on the notion of no free lunch, efficient markets and efficient portfolios. Modern Portfolio Theory is fond of expressing that your most expected outcome of playing the game is to achieve about the same as the average, and because of market friction, probably a little less.

Even the Stochastic Portfolio Theory of recent years will express its central theme of “growth optimal portfolio” as about achieving long-term average returns. It can be elaborated using stochastic equations that express the same thing as other acceptable theories: long-term prices can be expressed as a deterministic trend on which one will find a stochastic Brownian-like process. My **Jensen Modified Sharpe** paper uses these stochastic equations to explain the behavior of the trading methodology used.

What this leads to is that: the old Buy & Hold strategy can be improve and will relative ease. First one can adopt a Buffett-like style of investing. I have discussed this at more length in another article: **Growth Optimal Portfolio VIII**.

The conclusion of that article was: by reinvesting generated profits over the long haul would result in increasing the overall performance. The reason being that the generated profits would also earn some return instead of being left idle. Mr. Buffett over the life of his portfolio has achieved higher performance levels by reinvesting the generated profits from his holdings and also by simply holding on to his best investment choices.

It was easy to show Mr. Buffett's alpha generation. A chart of his performance compared to the S&P500 sufficed; it is reproduced below:

By reinvesting the generated profits in buying more shares, the above equation changes to:

$$[\text{ Capital}_t * (1 + g)^{(t-1)} / \text{Capital}_{t=0}]^{(1/t)} - 1 = r +$$

and therefore, simply re-investing the profits will generate a higher return. Both long-term and short-term players can do this. The short-term player by increasing his bet size while the long-term player by gradually buying more shares as profits increase and as time goes by.

With time, the short-term player will have a harder and harder time to flip his portfolio all the time and will encounter limiting factors to his/her growth. Too many trades to be made by hand at the same time, or too big bets on any one trade.

It is as if the short-term trader is doomed to remain relatively small. Not only does he start with a small stake; he is limited by his own resources and the speed at which he can type. Or, if putting his stuff on automation, limited by the randomness of price series. A long-term vision of the portfolio management problem needs to be looked at, otherwise, one is doomed to be part of the also ran.

By mixing approaches, meaning accumulating shares over time and trading over the share accumulation process would result in:

$$[\ Capital_{t} * (1 + g + T)^{(t-1)} / Capital_{t=0}\ ]^{(1/t)} - 1 = r{++}$$

And I think that the above equation is what can propel anyone to over-performance.

Overall Conclusion

What the Excel model 2 file showed it was sufficient to accumulate shares over an upward biased market to exceed Buy & Hold portfolio performance levels. Even if prices and trades were randomly generated, alpha was not only greater than zero, it was increasing in time. Not at a linear rate but at an exponential rate. All of modern portfolio theory with its no free lunch mantra is being questioned by a simple trading procedure which says: buy, accumulate and trade over the accumulative process. Re-invest the accumulating profits so that they can also earn some return.

If a randomly generated portfolio with randomly generated trades can show positive alpha and of an exponential type at that; a real portfolio using real market data should also provide positive alpha using the same trading principles by trading over a share accumulation program.

I have spent the last 18 months applying the above principles to actual market data. A number of trading strategies (taken from the old Wealth-Lab 4 site) were modified to incorporate my trading procedures with their underlying share accumulation programs. My web page has over a dozen of these simulations, all of which greatly out-performed their Buy & Hold counterpart.

All simulations were done over 6 years of data using the old Wealth-Lab simulator on their site using their market data. No leverage was allowed; a trade could occur only if there were funds in the account. It was like doing simulations in the cloud, on somebody else's machine. I would take a known trading strategy with its strengths and weaknesses and transform them to accumulate shares over time. The trading process then became a means to acquire more shares as the generated profits were put to use. And that is why all the simulations with these improved trading procedures far exceeded their original design. The methods used had a long-term view of portfolio performance; a multi-asset, multi-period and multi-strategy outlook of the future.

Guy R. Fleury

Thank you for your interest in my work.

